European Society for Engineering Education
Europäische Gesellschaft für Ingenieur-Ausbildung
Société Européenne pour la Formation des Ingénieurs

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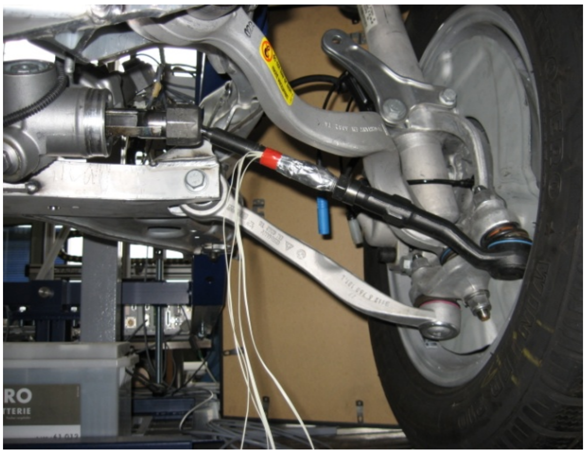
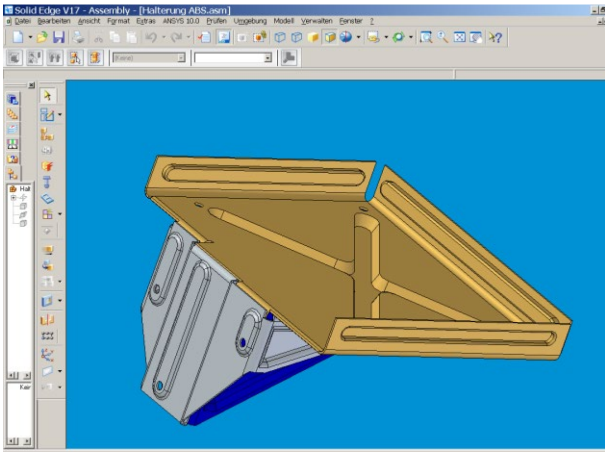
Making Sense of Engineering Workplace Mathematics to Inform Engineering Mathematics Education

A Report for the Mathematics Interest Group





Publisher: European Society for Engineering Education (SEFI)
Brussels 2021

ISBN: 978-2-87352-021-2

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Preface

In its fundamental curriculum document: "A Framework for Mathematics Curricula in Engineering Education", SEFI's Mathematics Interest Group uses the concept of "mathematical competence" to identify the goals of mathematics education in engineering study courses. Mathematical competence comprises the ability to understand and use mathematical concepts and procedures in relevant contexts and situations. The latter include mathematics-intensive application subjects like engineering mechanics or fundamentals of electronics within the study course but also the usage of mathematics in later work practice. Whereas information on application subjects can be gained within the university framework, it is much harder to investigate real workplaces. Most of the available studies on mathematics at workplaces are concerned with professions where an academic education is not required. There are not so many articles dealing with mathematics at engineering workplaces because it is not easy to really understand the work procedures, tasks and problems as an outsider. Moreover, the relevant publications are spread over different journals, books, and dissertations. This report is intended to provide the reader with an overview of available results from which it is easy to go deeper into single studies. It is hoped that it will inspire further investigations in a very under-researched, yet important area.

It remains to thank SEFI's Mathematics Interest Group, particularly the members of the Steering Committee (D. Velichova (Chair), M. Demlova, B. Olsson-Lehtonen, D. Dias Rasteiro, T. Gustafsson, D. Lawson, M. Brekke), for the opportunity to publish this document as a report for the Special Interest Group. Needless to say that for any error in this report the sole responsibility lies with the author.

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1. Introduction

In his introduction to a special issue of the journal "Educational Studies in Mathematics" in 2014, Bakker states that "the area of vocational education and workplace training has been chronically underrepresented in mathematics education research" (Bakker, 2014, p. 152). When it comes to engineering workplaces, the situation is even worse. The body of available research on using mathematical concepts, procedures and ways of thinking consists of only a few studies when one restricts engineering workplaces to those that require at least a Bachelor degree granted by a university. In this contribution I use this stricter definition, so I do not include technical qualifications e.g. of technicians which are also sometimes called "engineers" in literature. Given this narrower definition, virtually none of the studies in the most recent survey book (Bessot & Ridgway, 2000) was concerned with engineering workplaces. Since then, there were specific studies on structural engineers (Kent & Noss, 2002; Gainsburg, 2006, 2007, 2013), on industrial engineers (Cardella & Atman, 2005a,b; Cardella, 2006, 2010), on mechanical engineers (Alpers, 2006, 2010a), on electrical engineers (Romo Vasquez, 2009; Romo Vasquez & Castella, 2010; Hochmuth et al., 2014) as well as general studies on a broader spectrum of engineering (Goold & Devitt, 2012; v.d. Wal et al., 2017). The study by v.d. Wal et al. (2017) tries to extend results and concepts developed in studies on workers on an "intermediate level" (technically skilled workers) to the engineering profession, in particular the concept of "techno-mathematical literacy" which will be discussed briefly in the next section. Not all of these studies do actually investigate real engineering workplaces; some try to capture mathematics in engineering work by probing into the mathematical thinking of engineering students working on projects with practical relevance. The advantages and problems of such approaches are discussed in (Alpers, 2010b) and will be briefly summarized below.

This report is based on the studies mentioned above. It will try to give answers to the following questions:

- What are we looking for: What counts as "mathematics" in engineering work?
- How can we capture what we are looking for: Which research methods have been used and are they adequate?
- Which results do we have so far: Can a common body of insights be identified?
- Which conclusions can be drawn regarding the mathematical education of engineers?

In the final section I will summarise the results and provide a short outlook on potential directions of future research.

2. What are we looking for?

When studies investigate the relevance of mathematics taught at institutions (schools, universities) in workplace settings, "mathematics" is often identified with mathematical concepts and procedures (e.g. arithmetic, algebraic computations, solving equations, investigating functions). This has been denoted as "school mathematics" (Gainsburg, 2005) or – in order to include also university education – "curriculum mathematics" (Goold & Devitt, 2012). In studies on mathematics at workplaces that do not require an academic education, often a "gap" was found between "school mathematics" and really observable mathematical 'behavior' (Gainsburg, 2005, p. 13) resulting in questioning the usefulness of looking for "school mathematics" in general. For engineering workplaces, this would be problematic, since one should expect the explicit occurrence of mathematical concepts and procedures at such workplaces. Therefore, Goold & Devitt (2012) tried to investigate the use of "curriculum mathematics" by engineers in Ireland. In order to make this more specific, they modified de Lange's "assessment pyramid" for this purpose, as shown in figure 1. Three dimensions are used here: mathematical domains, usage type, and level. The mathematical domains seem to be problematic regarding engineering education since they are taken from the original pyramid developed for school mathematics. It is not clear how the areas treated in engineering mathematics should be mapped to this classification. Mapping one- and multidimensional calculus and differential equations all to the category "functions" seems to be rather misleading. So, although the idea of categorizing "curriculum mathematics" in a more systematic and comprehensive way seems to be reasonable, the specific arrangement needs more thought.

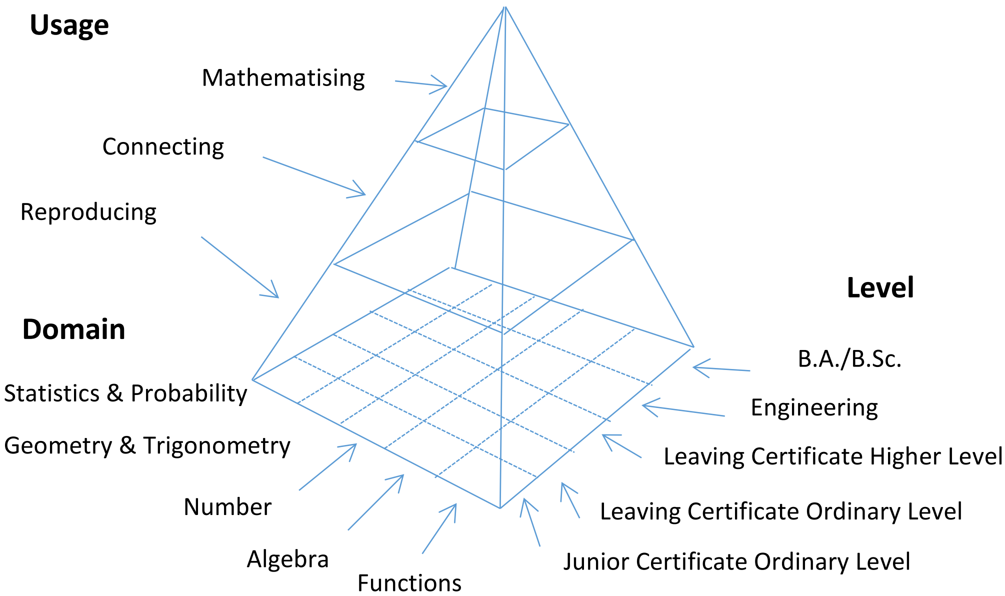

Figure 1: Curriculum Mathematics (after Goold & Devitt, 2012, p. 165)

It has been widely acknowledged that restricting oneself to observing direct occurrences of school or curriculum mathematics when investigating workplace mathematics results in too narrow a view of what counts as mathematics (Gainsburg, 2005). In order to broaden the view, notions like "mathematical behavior" or "mathematical thinking" have been used. The latter, specified in more detail by Schoenfeld (1992), is probably the most elaborated and influential concept. This was used by Cardella (2006, 2010) in her investigation of industrial engineering students working on their capstone projects. Schoenfeld identifies five aspects of mathematical thinking which are: the knowledge base (cognitive resources, facts and procedures); use of resources (planning and monitoring); problem solving strategies; beliefs and affects (beliefs about one's own capabilities, feelings towards mathematics); and mathematical practices (e.g. applying a mathematical perspective; mathematizing; dealing with uncertainty). It should be noted that one aspect of Schoenfeld's conceptualization takes explicitly into account the affective dimension of mathematics usage. This has also been used by Goold & Devitt (2012) in their research to capture the role of mathematics in engineering in Ireland.

Yet another elaborated way of conceptualizing mathematical behavior is provided by the concept of mathematical competence as specified by Mogens Niss (Niss, 2003; Niss & Højgaard, 2011). This has gained considerable influence via the OECD Pisa study series. It has also been used in the Curriculum Framework Document issued by the Mathematics Working Group of the European Society for Engineering Education (Alpers et al., 2013) and in a specific mathematics curriculum for a practice-oriented study course in mechanical engineering (Alpers, 2014). Niss defines mathematical competence very broadly as "the ability to understand, judge, do and use mathematics in a variety of intra- and extra-mathematical contexts and situations where mathematics plays or could play a role" (Niss, 2003). This is then specified in more detail by identifying eight so-called competencies which are: mathematical thinking; mathematical reasoning; mathematical problem solving; mathematical modeling; representing mathematical entities; handling mathematical symbols and formalism; communicating in, with, and about mathematics; making use of aids and tools. The competencies overlap but have a distinct "kernel". Looking for mathematical competence at the workplace would not only include the actual occurrence of such competence but also interesting instances of missing mathematical competence, e.g. when a mathematical approach "could have played a role". This takes into account the critique by Gainsburg (2005) that research emphasizing the gap between school mathematics and everyday mathematics "may reveal what is but does not tell us what is possible" (p. 9). No study has as yet explicitly used the competence approach for investigating engineering workplace mathematics. When conducting my own studies (Alpers, 2010a), I used a rather unelaborated concept of 'mathematical qualification' or 'expertise'. But using the competence concept would facilitate to connect research results to mathematics curricula which are also specified in terms of mathematical competence (see Alpers, 2014). Then one could even use a common concept comprising both "curriculum mathematics" (concepts and procedures) and "mathematical behavior".

Taking into account that many workplaces rely on the use of technology one way or the other, (Hoyles et al., 2010) specifically looked for what they called "techno-mathematical literacy" (TmL) in their study of mathematics at intermediate-level workplaces. With respect to engineering, v.d. Wal et al. (2017) used the concept of TmL for interpreting the results of interviews conducted with engineers. Hoyles et al. (2010) define TmL as the ability to "understand and use mathematics as a language that will increasingly pervade the workplace through IT-based control and administration systems as much as conventional literacy (reading and writing) has pervaded working life for the last century. This literacy involves a language that is not mathematical but 'techno-mathematical', where mathematics is expressed through technological artefacts" (p. 14). In engineering where the working environment (except for pure management positions) is full of technological tools (CAD, FEM, multibody dynamics simulation, machine element dimensioning programme, CFD, …) the mathematical competence necessary for using these tools in order to work on engineering tasks effectively and efficiently is certainly a major aspect to investigate. Nonetheless, the general mathematical competence approach does also include this aspect (cf. the competencies "making use of aids and tools" and "representing mathematical entities") but has a broader view.

A different approach to capturing mathematics usage is the one based on the "Anthropological Theory of Didactics" (ATD) developed by Chevallard which has been very influential in the French didactical community. There, the concept of "praxeology" has been introduced which comprises tasks and techniques (practical block) as well as technology and theory (theoretical block) (also called the "4T-model"). Here, technology is meant to be the explanation and justification of the used techniques, and theory is the theory this justification is based upon. A "praxeology" serves to describe and understand mathematical practices in an "institution" or community of practice. This has been used by Romo Vasquez to analyze the work of students in their fourth-year projects (Romo Vasquez & Castela, 2010; Romo Vasquez, 2009) and by Hochmuth et al. (2014) to compare the use of mathematical concepts in signal analysis and mathematics lectures and text books. The 4T-structure helps to identify differences between praxeologies and the place where they occur. Yet, it is not as specific as the elaboration of mathematical thinking by Schoenfeld or the elaboration of mathematical competence by Niss.

Instead of or in addition to taking an existing conceptualization of mathematical behavior to guide the investigation of workplaces, one might also try to develop categories of mathematics usage from the observation material itself. This is advocated by Gainsburg (2005, p. 15) who suggests that "researchers should allow new descriptive categories to grow from their observations and analyses". A good example for this are the notions of "boundary objects" or "breakdown situations" introduced by the group around Hoyles and Noss (see Hoyles et al., 2010; Noss & Kent, 2002).

3. How can we capture what we are looking for?

Regarding the second question stated above on how we can capture what we are looking for, we can distinguish between quantitative and qualitative methods. Only the study by Goold & Devitt (2012) applies quantitative measures among others in a mixed-methods approach. Their intention for doing so was to obtain data not just for a few 'test persons' but to measure the mathematics usage of the community of all 5755 chartered engineers in Ireland and to get not only qualitative but quantitative data for analyzing the relative importance of different parts of "curriculum mathematics" and "mathematical thinking" and the relationship with job roles and disciplines. A survey was conducted using a questionnaire capturing the "curriculum mathematics" pyramid depicted in figure 1. Engineers could mark on a Likert scale how often they used parts of "curriculum mathematics" resulting in 75 marks for the different combinations. Attributes are attached to the Likert scale for explanatory purposes (1: "not at all"; 2: "very little"; 3: "a little"; 4: "quite a lot"; 5: "a very great deal"). In order to get quantitative data also on "mathematics behavior" Goold & Devitt (2012) included questions on the usage of "mathematical thinking" and on beliefs and affects ("self-efficacy", enjoyment of mathematics, seeking a mathematical approach, …) using the same Likert scale. The questionnaire was sent to all chartered engineers and Goold & Devitt got 365 responses which is quite a good response rate. This quantitative approach raises several questions and concerns:

- Goold & Devitt assume that they have got a random sample of the set of Irish chartered engineers. They substantiate this claim by showing that – among others – different branches of engineering (civil, mechanical, electrical etc.) and different roles are represented properly. Still there remains the question whether the attitude towards mathematics does influence the willingness to answer such a large questionnaire. About 74% of the respondents state that they enjoyed mathematics in the last 6 months (p. 208) and about 80 % felt confident at it (p. 211) "quite a lot" or "a very great deal" (cross at 4 or 5 on the Likert scale). Given that there has been a strong concern about the mathematical capabilities of beginning engineering students at many places (including Ireland) during the last fifteen to twenty years which resulted in mathematics support measures (cf. Alpers et al., 2013, p.53ff), it is questionable whether the respondents were really representative for the community of Irish chartered engineers regarding attitude and capabilities. Goold & Devitt (2012, p.451) also see this as a potential limitation.
- When one wants to obtain quantitative data on the "role" of mathematics in engineering work, it might be problematic to just ask for the times of occurrence as was done in the Likert scale. Even if mathematical methods are used only once in a while they might be important for the overall success of a project. That has been observed by Goold & Devitt in the qualitative part of their study (interviews with 20 engineers) where it was stated that although mathematics takes up only 10% of the work this was an important part ("… that extra ten per cent that you actually get paid for at the end of the day", p. 373).
- In the Likert scale the numbers (1 to 5) are given additional meaning by providing a verbal explanation, presumably because the authors thought that the participants needed help in interpreting the pure numbers. It is not clear whether the choice of wording influences the behavior of the participants. One could, for example, expect "a little" to be attached rather to number 2; and there seems to be quite a difference between "a little" (=3) and "quite a lot" (=4).
- It has often been stated in empirical workplace studies (Hoyles et al., 2010, p. 8) that those being observed did not realize that they 'do mathematics' or 'behave mathematically' because this was deeply embedded in their work routines. Therefore, the question should be raised whether the respondents of a questionnaire are really able to answer questions on their

mathematics usage. Regarding the study by Goold & Devitt this applies in particular to the level dimension of the "curriculum mathematics" pyramid and to the questions regarding "mathematical thinking". Although there was some explanatory material given along with the questionnaire (but who reads this?), it is not sure whether engineers really understand the meaning of terms like "connecting" or "mathematising" developed by educational researchers. This is even more questionable for the term "mathematical thinking" for which the following list of examples is given as a result of interviewing engineers: "problem solving; 'big picture thinking'; decision making; logical thinking; estimation and confirmation of solution" (Goold & Devitt, 2012, p. 409). These are very broad explanations such that nearly every thinking process could be counted as mathematical thinking. Therefore, it is not surprising that it scored very high (over 4 on the Likert scale).

What kind of information can one expect to get from such quantitative data? In the study by Goold & Devitt most combinations ended up scoring a mean of somewhere between 2 and three ("very little" and "a little"). The single numbers do not give much information, it is rather the comparison of numbers that could give information on the relative importance. If the data capture is very detailed (as is the case in the study with 75 single results on "curriculum mathematics") this might be even counterproductive since it makes comparison with other data more problematic: When, for example, the importance of "curriculum mathematics" is compared with the importance of mathematical thinking by Goold & Devitt (see p. 252), they have the problem of "summing up" the data on the 75 categories. One could ask provocatively whether 75 times "a little" is very much. The authors have recognized this interpretive problem themselves (p. 252: "a score of 2.73 out of 5 for overall mean usage is interpreted as a high score"). Therefore, a very careful questionnaire design is necessary that takes into account the kind of information one wants to obtain from the results. In their study, the authors got results on the relative importance of single points in the pyramid (compared to others and to the somewhat vague "mathematical thinking") as well as on the relationship between roles (design/development; education; maintenance; management/project management; production) and discipline (civil; electrical; mechanical) on the one hand and importance on the other hand.

All studies on mathematics at engineering workplaces apply qualitative methods. These include the following ones:

- Engineers are observed when doing their daily work (see e.g. Noss & Kent (2002) and Gainsburg (2006) for structural engineers). This approach is called "ethnographic" research (see Hoyles et al., 2010; Zevenbergen, 2000). Zevenbergen distinguishes between real participation ("hard-core" ethnography) and 'mere' observation ("soft-core" ethnography). When it comes to engineering workplaces it will be hardly possible that the researcher can really participate because (s)he is not qualified for doing this. The observation is also somewhat restricted: Researchers can participate in meetings and observe discussions between engineers and learn from the problems and questions that come up there. But it is always the question whether they really are able to understand the problems. It is even more questionable if the researchers can see whether a more mathematical approach could be helpful (in the sense of "mathematical competence" defined by Niss as the ability to use mathematics in contexts and situations "where mathematics plays or <u>could play a role</u>" (underlining by B.A.)). This is discussed in more detail in (Alpers, 2010b).
- Given the problems of observing "real engineers" working on projects, some researchers investigated the work of students in the last year of their study course when they were given special projects (capstone projects, see Cardella dissertation 2006) or tasks that might occur in the daily work of a junior engineer (see Alpers, 2010a; Romo Vasquez, 2009; Romo Vasquez & Castella, 2010). The students could make use of industry strength tools such that in this

respect the working environment of a real engineer is taken into account. This allows for deeper probing into the thought processes because it does not cost the valuable work time of an engineer; on the other hand, there is not the usual company environment and the restrictions of daily engineering work are not present. The chances and limitations of this approach are discussed in more detail in (Alpers, 2010b).

- A further way to get information on the mathematics usage of real engineers is the (mostly semi-structured) interview. There, one can probe deeper into the mathematics usage as seen and recognized by the engineers themselves. Often the problem comes up that the engineers do not see their work as containing mathematical components because these are embedded in specific application contexts. Still, the restricted time frame for interviews (mostly 1-2 hours) leads to rather coarse insights, e.g. that "problem solving" or "software skills" are important.
- A quite promising way to learn more about the usage and role of mathematics is the investigation of artefacts produced in the work and used for communication between different communities. This has been proven very informative in the work by Hoyles et al. (2010) where for example specific process control data sheets acted as "boundary objects" between different communities in a company. In engineering, such artefacts can be for example any programme output (Excel© worksheets, CAD, FEM, machine element dimensioning programme, measurement data sheet) or any requirement specification that serves as communication interface between those who set it up and those who have to produce something fulfilling it. Other documents like engineering guidelines (general ones issued by engineering bodies or company specific ones) can also be considered here. When mathematical concepts or models are embedded in such artefacts or programmes it is interesting which understanding of the mathematical model lying behind it is still necessary for the effective and efficient use. This can be particularly important in so-called "breakdown situations" (Noss & Kent, 2002; Alpers, 2010a) where the usual routines do not work any longer and a look 'behind the interface' is required.

In Alpers (2010b) we state some general problems coming along with a qualitative approach. Since one investigates only a small sample of the diverse and heterogeneous engineering world, the question of whether it is possible to generalize the results comes up. The quantitative results by Goold & Devitt show that there is a dependence on role and discipline regarding mathematics usage. Therefore, qualitative studies should always make clear the boundary conditions of the investigation such that one can recognize potential limitations. It is also necessary to conduct several qualitative studies in different environments in order to get a more comprehensive picture. On the other hand can qualitative studies provide much deeper explanatory results for mathematics usage and thinking than is possible in surveys where it is always questionable whether the participants have the same understanding of a topic or a mark on a scale as the researcher. The categories developed so far to describe the kind of mathematical usage and behavior are helpful and can be used to inform education. The notions of "boundary objects", "(in)visible mathematics", "iterative working style", and "breakdown situations" help to recognize where mathematical concepts and thinking might show up or are buried behind an interface, and where a mathematical approach might be necessary to overcome problem situations.

Independent of the research method chosen, one problem will still be there: "Workplace Mathematics is a moving target" (Gainsburg, 2011, p. 117), so results may vary over the years such that a simple aggregation of scientific knowledge over decades might not be possible.

For investigating mathematics in engineering workplaces an interdisciplinary approach is particularly promising. Ideally, the research would be conducted by a team consisting of an engineer, a mathematician, and a mathematics education researcher which can hardly be found in one person.

Having an engineer involved makes it possible to get information on how representative the observed behavior is. For example, in my own studies with final year students (Alpers, 2010a) I collaborated with a colleague from engineering to set up ‘realistic’ tasks for the students resembling tasks for junior engineers. Moreover, discussion of the results with this engineer revealed that the procedure performed by the students was typical for a specific situation in engineering: The students started with an initial design and then performed some goal-oriented iterations to come to an acceptable solution. An alternative would have been to set up an optimization model and perform mathematical optimization. Whereas the former is the usual procedure when creating special machines (low numbers), the latter is often used in serial production where small gains (“the last 5%”) are important. Here, having a mathematician is helpful in order to get an idea of mathematical alternatives, and the role of an engineer would include a judgment on the often context-specific suitability of the observed approach. Having an engineer makes it also possible to detect flaws in engineering work. Otherwise, there is the danger that the observed practice is uncritically considered as “good practice” although a different, maybe more mathematical approach requiring modelling competence, might have been better. In the study by Gainsburg (2006), for example, structural engineers discussed and finally used a model although it did not seem to fit properly (but had the advantage that it was acknowledged such that one was “on the safe side” when referring to it); it remains unresolved whether they could have come up with a more suitable model if their modeling competence had been better. Having an engineering colleague in the team has the additional advantage that (s)he can be questioned more deeply and more often (e.g. when new questions come up after a while) whereas availability of engineers at work is rather limited.

The competence of an educational researcher is certainly needed for applying the qualitative and quantitative research methods and for relating the research to existing educational research. The theoretical and methodological issues discussed above definitely belong to this area: What counts as “mathematics” and how can it be captured? Educational researchers are also important when it comes to drawing conclusions from the research results regarding the mathematical education of engineers: Which learning scenarios are adequate for preparing engineers for making proper use of mathematics at later workplaces (or in application subjects). The article by Cardella (2008) can serve as an example for drawing such conclusions.

The considerations presented above show that an interplay of the three roles could be very productive. The major challenge presented by such a scenario is to get a common understanding as far as this is necessary. It is neither possible nor required that the engineer or the mathematician fully understands the educational research methods or that the educational researcher fully understands the engineering problem and the mathematics used in there. The challenge rather consists of finding an adequate “interface” (“boundary objects” in the sense of Hoyles et al., 2010) across which a meaningful communication is possible. The author experienced a similar situation when cooperating (as a mathematician) with engineers when writing engineering guidelines for the German Association of Engineers (VDI).

4. Which insights have been gained so far?

Regarding the direct usage of “curriculum mathematics” Goold & Devitt (2012) provide quantitative data across engineering branches and job profiles based on their survey with 365 Irish engineers. As already discussed in the previous section, this information is necessarily coarse and must be interpreted with care. On the average, mathematics usage scored with 2.73 on a five-point Likert scale between 2 (“very little”) and 3 (“a little”) (p. 408). They found out that the average value depends on discipline (branch) and role in a company but could not detect more specific mathematics content profiles depending on branch and/or role. Of particular importance are “statistics and probability” in order to analyse and interpret data for making decisions (p. 388). Based on their interviews Goold & Devitt (2012, p. 434) conclude:

> “One key message about engineering practice that emerges from the study is summed up by one engineer who presents that in a “typical engineering company” only “a few people” do “maths at quite a high level”, there are “people below them who need to understand and interpret what they are doing and then others who just need to know the big picture””.

Qualitative studies can give more detailed insights into used mathematical concepts but cannot be easily generalized across branches and profiles, so they provide rather ‘spot-lights’. In my own studies (cf. Alpers, 2010a) I listed several mathematical concepts students used when working on realistic tasks using state-of-the-art computer programs. The concepts were often directly connected with application meaning (velocity function, force vector) and this kind of embedding was also recognized by Kent & Noss (2002).

Looking for direct occurrence of mathematical contents provides just one facet of mathematics usage; other ways of acting mathematically or being able to do so have been conceptualized as “mathematical thinking”, “mathematical competence” or “techno-mathematical literacy” as explained in the first section. The quantitative investigation by Goold & Devitt (2012) showed that the usage of “mathematical thinking” (comprising “problem solving, big picture thinking, decision making, logical thinking, estimation and confirmation of solution”, p. 385) scored significantly higher with a value of 4.19 on the Likert scale and was independent of discipline and role. In the sequel I use the framework of mathematical competencies (cf. Niss, 2003; Niss & Højgaard, 2011) for ordering and presenting the currently available research results. This approach makes it easier to draw conclusions for educational practice since the same framework is often used in educational settings, particularly in the curriculum framework document issued by SEFI (Alpers et al., 2013). One should keep in mind that if aspects of competencies do not show up because they have not been observed, they might nevertheless be important for efficient and effective work.

<u>Thinking mathematically</u>: Understanding and judging what kind of questions can be answered using mathematics and hence where a mathematical approach might be helpful.

Using Schoenfeld’s five aspects of mathematical thinking for the analysis of students working on realistic projects, Cardella & Atman (2005a) concluded that “having a mathematical perspective and a mathematical vocabulary” was important for successful work. Based on her workplace studies, Gainsburg (2007) promotes “skeptical reverence” as adequate attitude of engineers towards the role of mathematics: mathematics is not the ‘golden bullet’ for solving all engineering problems but it is a useful tool. So, formulating a problem in mathematical terms can be helpful but other non-mathematical issues are also important and have to be taken into account. (Goold & Devitt 2012) include the affective domain in their analysis comprising self-efficacy and motivation to try a mathematical approach. They found in their interviews that “confidence in mathematical ability and in mathematical solutions are the main motivators for engineers to use mathematics in their work” (p.384), and it also depends on the value given to curriculum mathematics in a company whether or not a mathematical approach is followed (p. 385, 413 ff). They recognized an interesting difference between the view of mathematics at work and in education: "In engineering practice mathematics is used primarily as a tool to estimate and confirm multiple solutions to real problems while in engineering education mathematics is about deriving a unique and exact solution to theoretical problems from first principles." (p. 390, 422). It might be questionable, though, to which extent this description of mathematics education is really valid and not just the interpretation of former experience by the interviewed engineers.

Reasoning mathematically: Understanding a mathematical argumentation as well as setting up an own mathematical argumentation.

As opposed to what is familiar as reasoning in mathematics as a science where theory is built upon axioms and structures of assertions deducted from them by proof, the mathematical reasoning in engineering practice is rather 'fragmented', taking (tacitly) certain assertions/properties as given and arguing based on those. This was found in (Alpers, 2012a) where students working on realistic tasks argued why a certain boundary curve of a cam disk should be part of a circle or why the acceleration function as derivative of the velocity function should not oscillate erratically when the velocity function did not. Mathematical reasoning was also found regarding the influence of parameters on other quantities when one has to dimension machine elements in order to have a certain effect on the maximum stress they can bear. Reasoning is mostly combined with application meaning: a place where a part under load is fixed should have higher stress around it (Alpers 2012a). So, mathematical and application reasoning are strongly connected, using qualitative application rules (like put more material at places with higher stress values) which are based on calculations in simple models but might have become disconnected after a while. Romo Vasquez (2009) and Hochmuth et al. (2014) use the Anthropological Theory of Didactics to identify different praxeologies in mathematics and application subjects but have not yet extended this to engineering practice. Goold & Devitt (2012) identified in their interviews "logical thinking" as one constituent of "mathematical thinking" but this is not further elaborated.

Posing and solving mathematical problems: Formulating a question as a mathematical problem and using problem solving methods and strategies.

What constitutes a mathematical problem is not well-defined and probably depends on the capabilities of the person who should solve it. Using routine procedures for standard problems was observed by Gainsburg (2007) and Alpers (2012a) where textbook algorithms were used to dimension machine elements. In (Alpers, 2012a) problem solving in geometric design work appeared to be more challenging: Given certain geometric operations, how to proceed to create the imagined desired shape of a machine part. One strategy here was to start with a comprehending block and remove parts successively. Since this is done using CAD technology this might be considered as an aspect of techno-mathematical literacy since technology affords specific working styles of trial and error. V.d. Wal et al. (2017) correspondingly stated "technical creativity" as one aspect of techno-mathematical literacy they identified based on their interviews. In (Cardella & Atman 2005b) two problem solving strategies were recognised: "guess and verify" and "separate larger problems into smaller ones". In (Alpers 2012a) I identified in design work the strategy to start with an initial design (often based on rough models) and iterate meaningfully by using qualitative knowledge. As already stated in the previous section, the adequate way of problem solving might depend on the situation: in 'special machinery' work one wants to create quickly a feasible and acceptable design which might be sub-optimal, whereas in 'serial production' there is an incentive to go much further into optimization in order to exploit the 'final 5%' in potential. Goold & Devitt (2012, p. 387) also mention speed of response as important criterion stated by engineers in interviews. In general, they identified "problem solving" as essential part of "mathematical thinking" but this is not further specified and elaborated.

Modelling mathematically: Understanding, working and solving problems within models set up by others as well as performing active modelling.

This competency has been elaborated in more detail by specifying the so-called modelling cycle which models the modelling process and gives rise to the following sub-competencies (cf. Kaiser & Brand, 2015): understanding the situation and problem; setting up a real model by structuring, making assumptions and simplifications; setting up a mathematical model (mathematising); working mathematically; interpreting the results; validating the results; exposing the results to interested parties; planning and monitoring the overall process called "overall modelling competency" in (Kaiser & Brand 2015). All of these sub-competencies have been identified in the research literature. Goold & Devitt (2012, p. 390) cite an engineer stating that the engineer's role is "to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate that to the decision maker".

In more detail, (Gainsburg 2006, 2007a) found that "understanding the phenomenon" was a major effort in the engineering work she observed. In the interviews conducted by Goold & Devitt (2012, p. 410) an engineer stated: "engineering should be about trying to identify the right question, because a lot of times, people are obsessing over the wrong question".

Regarding setting up a real model Gainsburg (2005) observed both the creation of new models and choosing among existing ones. Observing students working on realistic tasks I identified modelling activities using well-known modelling quantities and principles as major activity (Alpers, 2010a). Both Cardella and Gainsburg emphasise the importance of estimation and dealing with uncertainty which comes up when one makes assumptions and simplifications.

Since the real world models are already stated in mathematical terms (e.g. statics models consisting of equilibrium equations) the mathematisation is essentially already performed when setting up real models. Working mathematically includes standard as well as more challenging problem solving which was already described above.

Regarding the importance of interpretation Goold & Devitt (2012) found that "Engineers say that engineering problems have multiple answers and that their job is to determine "what the answer means", which is "the best answer for all participants" and what "is the knock on effect" of the answer" (p. 409). V.d.Wal et al. (2017) identified having a "sense of number" as aspect of techno-mathematical literacy which is also related to interpretation. Even when engineers work mathematically, the concepts remain embedded in applications, so application meaning never vanishes.

Regarding validation v.d.Wal et al. (2017) identified a "Sense of error" as aspect of techno-mathematical literacy meaning that an engineer should be able to "check and verify data and detect errors" but this does not provide information on how this can be done. In my own studies (Alpers 2010a) I found that building up expectations based on basic models and mathematical properties helped engineering students working on realistic tasks to detect questionable results and tool output. Cardella (2010) observed discrepancies between (model-based) simulation and measurements as a source for thinking about validity. Such situations where inexplicable results show up have been called "breakdown situations" (Noss et al., 2000) and they often require to go deeper into the modelling process and to look behind interfaces in order to be able to explain the results. But it is not just mathematical arguments playing a role in such a situation. Gainsburg (2007) reports that engineers retained a not well-applicable but otherwise well-accepted standard model because this would make it easier to justify their decisions. Goold and Devitt (2012) emphasise that engineers have to take into account “real world practicability”.

Exposition includes justification and communication of results often in order to arrive at a consensus. This will be taken up when dealing with the communication competency.

Regarding the “Overall modelling competency” Gainsburg (2006, 2007a) identified “keeping track” as a major challenge. Cardella & Atman (2005a, b) observed planning and monitoring as aspect of “mathematical thinking”. In my own studies based on communications with an engineering colleague I recognized that engineers have to take into account many surrounding aspects (costs, availability of parts, logistics, possibility to produce and mount a part). Goold and Devitt (2012, p.409) cite the phrase “bigger picture thinking” in this respect.

<u>Representing mathematical entities</u>: Choosing adequate representations and switching between representations in order to use the most suitable one for a problem.

Interpreting data and graphical representations is called “data literacy” by v.d.Wal et al. (2017) which was identified as aspect of techno-mathematical literacy. Technology plays a role when it comes to changing representations e.g. when turning a value table into a graph using a spreadsheet program in order to better recognise developments. V. d. Wal et al. (2017) also consider the ability to understand technical drawings as important component of TmL. In my own studies (Alpers 2010a) I investigated the representations provided at software user interfaces since these are the boundary objects between user and technology. Most often the representations were graphical ones with properties for which data had to be specified.

<u>Handling mathematical symbols and formalism</u>: Understanding symbolic expressions and formal language used by others as well as setting up own ones.

In my studies it was quite evident that CAD programme designers avoided symbolic (algebraic) expressions whenever possible and used geometric objects, properties and operations. On the other hand, when students used machine element dimensioning software they made computations by hand in small models in order to validate their results such that they not end up being a ‘slave’ of the tool. For this they used formulae and algorithms presented in wide-spread textbooks on machine elements. Moreover, the user has to transform output from one part of the programme into input for another one requiring computations by hand. In motion design tasks like the one my students worked on one can use a guideline of the German Association of Engineers which is full of symbolic representations of functions mostly put into a normal form on the interval [0,1]. Cardella & Atman (2005a,b) observe the use of formulae in Excel and symbolic computation within models.

<u>Communicating in, with, and about mathematics</u>: Understanding oral and written mathematical statements made by others and expressing oneself mathematically.

Goold & Devitt (2012) emphasise the importance to communicate solutions gained using mathematical methods, in particular the communication with non-experts (p. 388: “… putting the mathematics “into a form that a non-engineer will understand””). They identified different forms of communication:

“Engineers communicate mathematics when: expressing engineering concepts; expressing conclusions; writing reports; making arguments; explaining how “you have come to your conclusion”; justifying some decisions; rolling out IT solutions; reading reports; verifying consultants’ work; communicating a concept to a decision-maker; asking finance people to provide money; and selling products.” (p. 434).

Similarly, v.d. Wal et al. (2017) identified “Technical communication skills” as another aspect of techno-mathematical literacy. Gainsburg (2013) also states that engineers have to explain and justify their decisions. In my own studies I also found the requirement of communication between different engineering departments, e.g. measurement and computation or design and computation (Alpers 2010a). It is important to identify the boundary objects at these interfaces. In case of mechanical engineering, measurement and computation departments have to share a common application model for strain, stress and loads. For being able to communicate meaningfully an engineer must have an idea about the interfaces in order to make statements the communication partner is able to understand.

<u>Making use of aids and tools</u>: Recognising when the usage of aids and tools is adequate as well as using them adequately.

There is no doubt about the great influence of technology on engineering work since software programmes can be found at any engineering workplace. Therefore, v.d. Wal et al. (2017) found in their interviews that “software skills” were the most frequently encountered aspect of TmL. They found “white box”, “grey box” and “black box” usages of software. Users have to at least understand the mathematical objects at the interface (boundary objects) but always in connection with application meaning. Understanding the underlying model is important for meaningful and efficient variation (e.g. machine elements, cf. Alpers, 2010a) or in breakdown situations; sometimes “understanding through use” (Kent & Noss, 2002; Alpers, 2006) might already be sufficient. If the software does not work as expected it is also important to be able to find workarounds (Alpers 2010a). Alpers (2010a), v.d. Wal et al. (2017) and Goold & Devitt (2012) emphasise the ability to verify and interpret output from software programmes since there might be a wrong understanding of both required input and delivered output. An engineer interviewed by Goold & Devitt states: “the engineer should understand how the program is solving the equations and what it is doing, because it is always dangerous not to’” (p. 411).

5. Drawing conclusions for educational practice

From their study Goold & Devitt (2012) draw the following conclusion: "The key message for engineering education is that building a mathematics curriculum that more closely represents the way mathematics is used in engineering practice will strengthen it." (p. 448). Regarding engineering education in general they state: "The findings in this study suggest that engaging in active or social learning environments that emulate engineering practice would benefit engineering education." (p. 449). But one has to keep in mind that the goals of the mathematical education of engineers are twofold:

- to enable students to work with the mathematical concepts, models and procedures used in the application subjects of the study course
- to enable students to work mathematically in later workplace environments.

For achieving the first goal the mathematical education should be strongly integrated in the respective engineering study course (see Alpers et al., 2013, p. 61-65, as well as Alpers 2020, for different aspects of integration). The mathematical work at later workplaces is strongly embedded in application contexts and problems and the mathematical concepts and procedures are often to a large extent hidden in software tools. Nonetheless, the available studies have shown that aspects of mathematical thinking (according to Schoenfeld, 1992) or mathematical competence (according to Niss, 2003) or techno-mathematical literacies (according to v.d. Wal et al. 2017) are required and should hence be addressed in the mathematical education of engineers. There are a few proposals for doing this in Cardella (2008), Alpers (2002, 2014a, b, 2015), Christensen (2008), and v.d. Wal et al. (2019). A "minimally invasive" and quite restricted attempt would be the inclusion of "authentic" tasks (possibly with a slight didactical reduction) in weekly assignment sheets (see Alpers, 2015 for an example concerning motion design). A more comprehensive approach includes group projects in mathematics education where students can gain experience on relevant aspects of mathematical thinking. This is most consequently applied in the "problem-based learning" approach to education (see Christensen, 2008). Cardella (2008, p. 157) recommends the use of 'model eliciting activities' which "are open-ended, real-world, client-driven problems based on the models and modelling perspective". Below, we state some project-based learning activities that are suitable for addressing aspects of mathematical thinking:

- Mathematical application projects as described in (Alpers, 2002, 2015) can be used to let students "understand, judge, do and use mathematics" (Niss, 2003) in relevant application contexts. The motion design problems described in (Alpers, 2015) occur in real engineering work, e.g. in the packaging machine industry. In such projects one can let students retrieve realistic data from data sheets available on the internet (on capabilities of machines like maximum acceleration of a motor) and do experiments in order to compare models with real data and thus validate models. Such projects might include the whole modeling cycle or work in already existing models. Students can learn how to deal with uncertainty and experience the necessity for estimation.
- In such projects students can also experience real practices like the iterative working style of many engineers (start with a rough initial design making educated guesses or using rough models; perform iteration cycles by changing variables in order to come to a "good" design). This kind of "mathematical practice" (Cardella, 2008) is often not included in mathematics education but important for real engineering work.
- One can also include the usage of tools that can be found at real engineering workplaces like Computer Aided Design (CAD) programmes or machine element dimensioning programmes which are available at universities for educational purposes. Students then see what they still

need to know mathematically when they use such programmes even if most of the mathematical models are ‘buried in code’. One can let them for example construct a parameterized design for a notch such that the geometric measures can be varied easily. For this one needs a design using geometric relationships which are preserved on variation (see e.g. Alpers, 2006). They can also implement an algorithm for dimensioning a certain machine element like a shaft or a spring and compare their results with professional machine element dimensioning programmes.

- In projects students can also experience the openness of real design work where there is not ‘one correct answer’ but an abundance of different solutions. In (Alpers, 2015) I show that motion design problems are particularly suitable for encountering such a situation when working mathematically.
- In projects students have to manage their resources (ask experts, plan work, get data, sub-divide the task, delegate sub-tasks to team members, use software etc.). They also learn to communicate mathematical concepts and practices to their team members and to other students in project presentations.

Van der Wal et al. (2019) investigate teaching strategies for fostering techno-mathematical literacies. In their design research study, they set up an applied mathematics course for life science students where the students had to work on three “cases” related to their field of study. After working on their own, there was a feedback session where groups presented their results and the lecturer applied a questioning approach based on inquiry-based learning. An analysis of this approach revealed that the lecturer fostered the acquisition of techno-mathematical literacies by several strategies including asking “students deeper questions about data, tables, formulas, and figures”, asking “students to elaborate on the answer”, and letting “students discover their mistake by stimulating thinking about the logical answer”. They conclude that the “feedback hours, with their classroom discussions, and usage of IBL questions, seem to contribute to the learning of TmL”. (v.d. Wal et al. 2019, no page numbers).

Regarding the affective dimension of learning, it is the goal for the students to develop a certain degree of consciousness of what they can do (and when they should ask an expert), a critical appreciation of the role of mathematics in engineering tasks (called “skeptical reverence” in Gainsburg, 2007), and a readiness to apply a mathematical approach (either by doing mathematics themselves or in cooperation with an expert).

Finishing this section, one ‘caveat’ should be stated explicitly. One should not expect university education to deliver ‘full grown’ engineers. As Gainsburg (2013) recognized in her study on the development of modeling competence from students to novices (recent graduates) to veterans, many aspects of this competence seem to develop over years of experience and work in many projects. Goold & Devitt (2012, p. 390) concluded from their interviews that “an ability to do engineering work comes from the “experience of working in an engineering environment” watching other engineers estimate, work out real problems and how they view “the bigger picture””. There also might be a development from a larger use of curriculum mathematics to a less one when advancing to management positions (Kent & Noss, 2002; Goold & Devitt, 2012). University education can provide opportunities to experience the different aspects and to induce meta-cognitive thoughts about the processes involved but it cannot anticipate years of experience in real engineering projects.

6. Summary and outlook

Compared to the state-of-the-art volume edited by Bessot & Ridgway in 2000 remarkable progress has been made in the still 'niche' area of engineering workplace mathematics. There are now a few studies concerning engineering workplaces using predominantly qualitative research methods where interviews play a major role. Capturing mathematics usage at engineering workplaces is not easy because different qualifications are needed. Therefore, interdisciplinary approaches including mathematics educators, engineers and educational researchers are most promising. A further problem is that there is hardly "the" engineer but there is a range of branches and job profiles with different requirements. But several results in different studies point in the same direction. In this contribution I structured the results using the framework of mathematical competencies in order to facilitate to draw educational conclusions. Some ideas for doing the latter are also presented but sometimes research results are too general (like "software skills") making it impossible to draw concrete conclusions.

From the findings stated above, it is clear that much more research is required to better cover the breadth of branches and job profiles in engineering. For being able to set up corresponding educational profiles a further investigation of boundary objects at different interfaces would be helpful, e.g. between sales engineers and customers or between different departments in companies: design and computation (a design department giving a task to a computational one and interpreting and discussing the results). Also a further look at the boundary objects at technological interfaces is required to better understand which models a user is still required to know to work effectively and efficiently with tools. Another route of research would be to conduct longitudinal studies on competence development over time as suggested by Gainsburg (2013). This could help to specify a reasonable split of responsibility between mathematics education, application education and informal lifelong learning at work.

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