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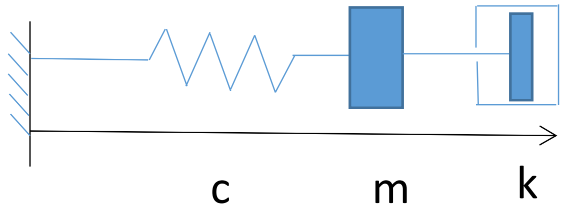
Mathematics as a Service Subject at the Tertiary Level

A State-of-the-Art Report for the Mathematics Interest Group



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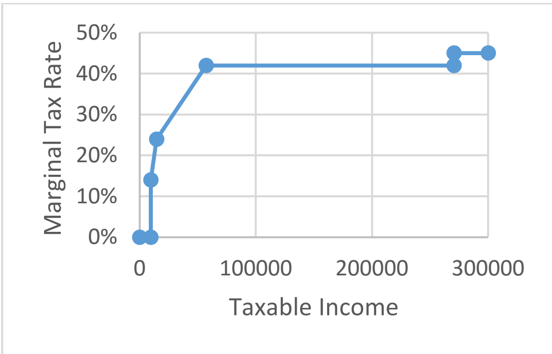
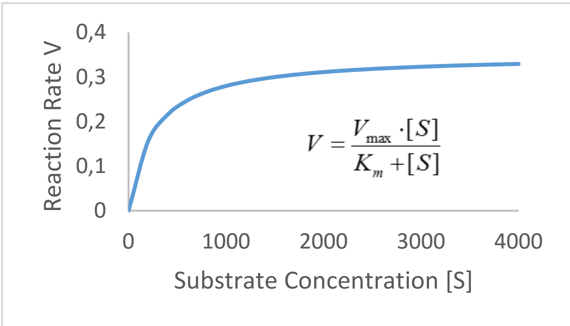


Type of offering	Mean Use (1-6)
Lecture	5,70
Tutorial	5,40
Assignment	5,26
Additional Tasks	3,72
ELO	2,01
Tests with feedback	1,82



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SEFI is the largest network of higher engineering education institutions (HEIs) and educators in Europe. Created in 1973, SEFI is an international non-profit organisation aiming to support, promote and improve European higher engineering education, enhancing the status of both engineering education and engineering in society.

SEFI is an international forum composed of higher engineering education institutions, academic staff and teachers, students, related associations and companies present in 48 countries. Through its membership and network, SEFI reaches approximately 160000 academics and 1000000 students. SEFI represents more than 4 decades of passion, dedication and high expertise in engineering education through actions undertaken according to its values: engagement and responsibility, respect of diversity and different cultures, institutional inclusiveness, multidisciplinary and openness, transparency, sustainability, creativity and professionalism. SEFI formulates ideas and positions on engineering education issues, influences engineering education in Europe, acts as a link between its members and European and worldwide bodies, contributes to the recruitment of good students whilst always promoting an international dimension in engineering curricula.

Our activities: Annual scientific conferences, annual conventions for engineering deans, ad hoc seminars/workshops organised by our working groups and special committees, scientific publications (incl. the bi-monthly European Journal of Engineering Education), European projects under ERASMUS + and Horizon2020, position papers, European debates, cooperation with other major European and international bodies. The cooperation with partner and sister engineering organisations in Europe and in the world is also one of our priorities.

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Preface

In 2013, the SEFI Mathematics Working Group published its curriculum document: "A Framework for Mathematics Curricula in Engineering Education". This document has since been cited in many publications and a Google search using "mathematics education engineering" provides about 247 million entries with the curriculum document appearing fourth (August 19, 2019). This shows that the document is widespread in the community of people interested in teaching and learning mathematics in engineering study courses. The framework document does not only provide information on setting up curricula using a competence-based approach but gives also an overview of important topics like transition, use of technology, assessment, and curricular integration.

In the last decade, there was a remarkable increase in research and practical interventions in this area. This was due to increasing problems with students not being mathematically well-prepared entering university study courses where mathematics is needed. Correspondingly, financial resources were made available to improve the situation by researching the teaching and learning processes and practices in more depth for finding facilitators of and obstacles to successful learning and trying out interventions like preparatory courses as well as new teaching/learning schemes like flipped classroom. In this development, privately and publicly funded centres like MatRIC in Norway, MEC in the UK, and KHDM in Germany played a major role in organising research and dissemination of research in conferences and networks. In the US, the RUME group played a similar role for mathematics education at the tertiary level but without a special interest in service mathematics. Regarding professional bodies, SEFI's Mathematics Interest Group (formerly: Working Group) and ASEE's Mathematics Division continued to organise conferences for mostly practitioners ("professional reports") to exchange ideas and report on interventions that gave insights into the conditions of successful learning.

Since after the issuing of the curriculum document so many new and deeper results have appeared in many conference proceedings and journal papers it seems adequate to provide a structured summary such that the community gets a quick, structured overview with pointers for further reading. This overview takes a broader perspective in that it tries to capture service mathematics in general although it is fair to state that mathematics in engineering education forms still the centre. It comprises on the one hand information on the historical development of the field and its major players, and on the other hand a brief account of important themes and corresponding results. There is only occasionally an overlap with the curriculum document such that it is recommended to read the latter first and then the present survey for being informed of the most recent results. This document does not claim to be comprehensive since there are so many developments around the world that would require much more coordinated effort to capture. The current report is certainly euro-centric but it is hoped that it will inspire reports from other regions of the world.

It remains to thank all the people involved in SEFI's Mathematics Interest Group, particularly the members of the Steering Committee: Daniela Velichova (Chair), Marie Demlova, Brita Olsson-Lehtonen, Deolinda Dias Rasteiro, Tommy Gustafsson, Duncan Lawson, and Morten Brekke. Needless to say that for any error in this report the sole responsibility lies with the author.

Contents

1. Introduction: What is “Service Mathematics” and what are its specific themes?

Mathematics education at the tertiary level has quite different goals depending on the kind of study course where it takes place. We can coarsely distinguish between three types:

- In mathematics and mathematics teacher education study courses students are introduced to mathematics as a scientific subject with its specific epistemology.
- Situated within a general basic education at college, mathematics education is rather generally oriented to provide an introduction to the field which is in several countries already part of upper secondary education.
- Situated in application study courses, mathematics education serves the purpose to enable students to acquire the mathematical competencies required in the study course as well as in the corresponding professional practice. Such application study courses comprise natural sciences (physics, chemistry, biology, …), engineering (mechanical, electrical, civil, …), economics/business as well as other arts and sciences where predominantly statistical methods are used (medicine, psychology, pedagogics, …).

Service Mathematics refers to the third type. It describes an offering that is explicitly made for students in non-mathematical study courses; it is not part of a general undergraduate education which is not specifically directed at such a study course, and not directed at mathematics or mathematics teacher students. This survey addresses the didactics of service mathematics. It first outlines the historical development starting with a study conference of the International Commission on Mathematical Instruction (ICMI) on this topic in 1987. The major subsequent ICMI activities are stated and the development as a research field within the scientific community is sketched. Furthermore, the activities of professional societies directed at education of professionals are discussed. In the main part of this contribution, the major themes for practice, research and development are identified mostly based on research and practice reported in the last 10-15 years such that the reader gets a brief overview of the state-of-the-art. Nonetheless, it should be noted that only a coarse, non-comprehensive description is achievable which should enable the reader to deepen the study of the field in a structured way.

2. Historical development and major players[1]

Already in 1911, the International Commission on Mathematical Instruction (ICMI) organised a meeting on “What mathematics should be taught to those students studying the physical and natural sciences?” (after Clements et al. 1988). Yet, it took another 75 years before the first ICMI study on “Mathematics as a Service Subject” was launched resulting in two corresponding books by Howson et al. (1988) and Clements et al. (1988). The discussion document (Howson et al. 1988) structured the discussion according to the three questions “Why”, “What?” and “How?”. Answers to “Why?” should specify the aims of mathematical education within the framework of the surrounding non-mathematical study course. Possible answers given in the document include

- Enable students to use techniques and concepts in application subjects
- Enable students to make use of technology, have a mathematical “mode of thought”

[1] Parts of this section are simultaneously made available as contribution to the ICME 2020 conference, see (Alpers 2020)

- Enable students to "read" mathematics for continuing professional development (learn mathematics as a language).

Answers to "What?" are certainly influenced by answers to "Why?". They include statements on the subjects that should be dealt with in a certain type of application study course, as well as an elaboration of the emphasis put on techniques and on "mathematical thinking". Answers also depend on the assumed prerequisites of students and on the person who teaches (mathematicians, experts of application study course). Answers to "How" are concerned with the mathematical rigor applied (statements and proofs), the role of examples and context, the use of resources like books (and other written material) and technology, and the role of assessment and other boundary conditions like class size.

The participants of the study conference were mainly reflective practitioners reporting about innovative arrangements they tested and corresponding results. Their experience was summarised in the book by Howson et al. (1988) as follows:

- Service mathematics is a "very varied, very interesting and ill-understood activity" (p. 5) which is often not considered as particularly rewarding.
- In order to answer the posed questions meaningfully close cooperation with colleagues from the surrounding application study course is absolutely necessary since only they know what mathematics is used and how it is used. Such cooperation can go as far as common class teaching.
- Besides specification of mathematical subjects to be taught the goal of acquiring "modes of thought associated with those subjects" is important. These "modes of thought" (elsewhere in the volume also called "mathematical thinking") are not well-elaborated in the document but examples are given ("various modes of expression: abstract exploration, geometric representation, an intuition into the calculus, then logical deduction and formal rigour" (p.7). The requirement of "mathematical literacy" is also explicitly stated which comprises the ability to read mathematics, to express oneself mathematically, and to communicate with mathematicians.
- The importance of examples taken from the surrounding study course and of modelling activities is also stressed. Students can experience the value of their mathematical education and are better motivated. Technology offers support in setting up and computing more advanced models and in working more actively and experimentally.

The discussion document and the participants' contributions show that many aspects of the current discussion and research were already recognised some 30 years ago. The "drivers" at that time were mainly reflective practitioners, so there was no embedding into didactical theory and no scientific analysis of didactical interventions. Concepts like "mathematical modes of thought" were used but not well-elaborated. Requirements like enabling students to use their mathematical education in application subjects were stated but there was no systematic research on how to capture the required competences. Likewise, the influence of technology on required mathematical techniques was already recognised but the question of which mathematical abilities are still necessary to make reasonable use of technology embedding mathematical concepts was not yet posed and researched explicitly. The participants also realised the importance of the background and motivation of lecturers but the influence had not been investigated systematically.

In ICME conferences which are organised by ICMI, service mathematics has usually been dealt with in working groups on mathematics on the tertiary level (see IMU webpage: https://www.mathunion.org/icmi/publications/icme-proceedings-and-publications). In 1988, there was an additional group on "Mathematics and other subjects" where mathematics as a service subject was addressed. Only in 1996, there was a special Working Group dedicated to “Mathematics as a service subject”. According to working group reports, both in 1988 and in 1996, mainly practitioner reports on didactical innovations were discussed. In 2016, service mathematics was part of the so-called Topic Study Group on Mathematics on the tertiary level. The respective contributions (see Kaiser 2016) as well as the State-of-the-Art Report issued in advance (Biza et al. 2016) demonstrate a remarkable shift to scientific investigations regarding the questions already stated in the ICMI report 30 years ago. This progress will be outlined in the next section.

Other ICMI activities related to the field under consideration include the ICMI study on “The Teaching and Learning of Mathematics at University Level” published in 2001 (Holton 2001) and the ICMI study on “Educational Interfaces between Mathematics and Industry” (Damlamian et al. 2013) which was preceded by a study conference documented in (Araujo et al. 2010). The first study contains a smaller section on “Mathematics and other disciplines” which consists mainly of practitioner reports on making mathematics more relevant or enhancing mathematical understanding. In addition, there is a contribution by Kent & Noss (2001) in the section on technology regarding the role of technology in service mathematics. They still state that “… Compared to the corpus of work on undergraduate mathematics as a whole, the teaching of service mathematics remains relatively unexplored, and many of the fundamental assumptions … remain unexamined.” They explicitly address the observation that in application domains mathematics is often invisibly embedded in tools which raises the important question which understanding is still necessary for making reasonable use of the tools (relationship between use and understanding). The study on educational interfaces is relevant for service mathematics since it addresses the question which mathematical competence is required in workplace activities in industry performed by graduates from application study courses. Since in these courses students are usually educated for later work in industry (in a broad sense), requirements from industry are quite important. Regarding engineers, general methodological questions are discussed by Alpers (2010) whereas Romo Vasquez & Castella (2010) report on their investigations on capturing the necessary mathematical qualifications using ATD (Anthropological Theory of Didactics) in order to identify mathematical “praxeologies” in advanced engineering projects performed by 4th year students and by comparing the treatment of Laplace transforms in mathematics and automation textbooks. Hoyles et al. (2013) take up the observation that mathematics is often invisibly embedded in tools and artefacts: “Information and communication technologies have introduced further layers of invisibility between employees and the mathematical models embedded within computer systems used as part of routine work practice.” (p. 44). In order to describe the still necessary mathematical competence they introduced the notion of “techno-mathematical literacy” the components of which are identified by workplace studies and developmental research on tools for better understanding “boundary objects”. In their conclusion of the study, Damlamian et al. (2013) state that “a balanced mathematical modelling education is the most important educational interface between mathematics and industry” (p. 451).

More systematic research on undergraduate mathematics provision in general and service mathematics in particular can be observed for about 20 years. In the US, this development was supported by setting up the MAA Special Interest Group on Research in Undergraduate Mathematics (RUME) which offers a forum for exchange at annual conferences. Most contributions to RUME conferences, though, deal with general aspects of undergraduate mathematics provision and are not specifically dedicated to service mathematics. In Europe, the European Society for Research in Mathematics Education (ERME) set up a thematic working group on “Advanced Mathematical Thinking” in 2005 which was reorganised as working group on “University Mathematics Education” in 2011 (see Nardi 2017, for the development). In the latter, service mathematics plays an increasingly important role (see Gonzales-Martin et al 2017). This is particularly due to the fact that in some European countries, competence centres were set up where the investigation of service mathematics is explicitly addressed, to name: The Mathematics Education Centre (MEC, UK), the Competence Centre for University Mathematics (KHDM, Germany), and the Centre for Research, Innovation and Coordination of Mathematics Teaching (MatRIC, Norway). The proceedings of the KHDM conference in 2015 (Göller et al. 2017) as well as the proceedings of the INDRUM (International Network for Didactic Research in University Mathematics) conferences in 2016 and 2018 (initiated by ERME) and the ICME-13 Topical Survey on "Teaching and Learning Mathematics at the Tertiary Level" (Biza et al. 2016) demonstrate that service mathematics is well recognised as an important but still under-researched field of university mathematics education as Artigue states in her overview presentation (Artigue 2016): „… the diversity of forms of professional relationship with mathematics for which university courses may prepare graduates is still not sufficiently investigated and taken into account." A first attempt to include the mathematics provision for non-mathematics majors can be found in the handbook article by (Artigue, Batanero, Kent 2007) where engineering mathematics and statistics education are dealt with explicitly. The next section provides an overview of the important themes of research and development in the area of service mathematics.

Regarding engineering education, service mathematics is also a topic of major interest in professional bodies like the European Society for Engineering Education (SEFI) and the American Society for Engineering Education (ASEE). In both societies, there are special groups working on the mathematical education of engineers. These groups intend to foster communication among practitioners and to provide orientation for those who are interested in the topic. For this, they provide seminars and conferences where problems and innovative ideas and settings are discussed (for SEFI see http://sefi.htw-aalen.de ; for ASEE see the mathematics division pages at http://www.asee.org). The themes presented and discussed at those events include integrating mathematical education into the surrounding study course, motivating and activating students using innovative learning arrangements like flipped classroom, the role of technology and transition, i.e. those topics that were already identified in the ICME study 1988. The contributions partially take into account research literature and apply established research methods but there are also many "reflective practitioner" reports which can inspire other practitioners to try out similar arrangements (called "professional literature" by Treffert-Thomas & Jaworski 2015). The SEFI Mathematics Working group has also developed a curriculum document which in its latest edition is based on the concept of mathematical competence (Alpers et al. 2013). This document is meant to provide a framework for specifying curricula for specific types of study courses as well as a brief account of important themes and results.

For other kinds of study courses (physics, other natural sciences, business studies, …) there do not seem to be institutionalised efforts by professional societies. Therefore, only occasional research projects and practice reports can be found (see e.g. the RUME and INDRUM conference proceedings).

3. Important themes for research and practice

In this section, we outline the important themes of service mathematics didactics as they can be found in contributions to practitioner conferences as well as in conference proceedings and journals where didactical research is published. The themes are grouped into five topics: Integration of mathematics education into the study course; learning resources and arrangements; transition issues; assessment; and teachers and teaching practices. With some themes, there exists a considerable overlap with general mathematics education at tertiary level. For example, flipped classroom arrangements or use of technology are discussed and investigated for all kinds of mathematics education. Here, we restrict ourselves to those aspects that are specific for service mathematics. We give a short overview of the themes based on literature taken mainly but not exclusively from the last five years. The references provide pointers to the literature but we do not claim to give a comprehensive overview.

3.1 Integration of mathematics into the study course

Mathematical education should be an integral part of the curriculum in an application study course in order to justify the relatively large effort that is often spent for this part of the overall education. This justification is directed both at study deans responsible for a study course as well as students who want to see the relevance. The topic “integration” has many facets some of which will be outlined below.

Use of mathematical concepts in theory development in application subjects

Mathematical concepts, models, procedures and reasoning play a certain role in the theory development in application subjects. Uhden et al. (2012) emphasise for physics that it is often not possible to separate physical theory development from mathematics (as is done in the idealized model of the “modelling cycle”) since “mathematics penetrates into the construction of the physical concept itself” (p. 492). Similarly, based on an analysis of textbooks on signals and systems in electronics, Hochmuth & Schreiber (2017) (cf. also Hochmuth et al. (2014)) found that from start on the objects (signals) introduced were mathematical objects with physical meaning. For students to be able to draw on their mathematical education, it is important to investigate whether the usage of mathematics in application subjects is “in line” with the one students encounter in mathematics. This comprises the way of reasoning as well as notation and rules of usage. Hochmuth & Schreiber (2017) discovered interesting differences in “praxeologies” using the theoretical framework of ATD (Anthropological Theory of Didactics). When introducing the Dirac “function” the textbook authors rather used “plausibility arguments” drawing on visualisations like the approximation with rectangular signals with constant area. The underlying problem is that a mathematically correct introduction using distribution theory is not possible and is also not justified by later theory usage where only certain properties are needed (like the sifting property). Nonetheless, a reference to mathematical theory serves as a justification for the plausibility approach as well as a reference to successful usage in practice. Students have to ignore what is normally required in mathematical approaches they encounter in mathematics education.

Alpers (2017b) found a similar case related to so called “virtual displacements” δx in an analysis of statics textbooks (in chapters on “virtual work”) where the underlying mathematical theory (variation theory) is also much too advanced for a proper treatment in engineering and some rules for dealing with such displacements (similar to differentials) are given. The approach is – among others – justified by getting results in statics more efficiently than by using other methods. It is an open question whether there should be any further didactical reaction to this situation or whether it should be just addressed as done in the textbooks using plausibility arguments. There are other differences regarding vectors and differentials (cf. Alpers 2017b) where an explicit relationship between treatment in mathematics education and application subjects like statics is possible. Moreover, using the framework of ATD, there are not only different “praxeologies” in application subjects and mathematics education but also between mathematics education in application study courses and in proper mathematics study courses.

Gonzalez-Martinez & Hernandes Gomes (2017) investigate the use of mathematical concepts from calculus in textbooks on “strength of materials” using ATD. They found considerable differences in praxeologies where in engineering the usage of limits and relationships with the mathematical concepts of continuity and differentiability were rather avoided. Instead, differential relationships between functions (like load function, shear force function, bending moment function) were explained using simple examples (polynomials of degree 1 to 3) without reference to general relations. The latter is problematic because of the special role of point loads (forces, moments) which result in discontinuities of some functions. Alpers (2018) showed in another comparative textbook analysis of a mathematics and a statics textbook that there are differences in the definition of continuous functions in mathematics and in statics regarding the inclusion of single points on the real axis. Moreover, the concept image required for statics is rather restricted to functions consisting of continuous pieces as opposed to the more general definition used in mathematics. Regarding the use of integrals in a textbook on “mechanics of materials”, Gonzalez-Martinez & Hernandes Gomes (2018) realised that integral formulae for computing static moments and centres of areas are stated but not used further on since for standard geometries the results are made available in form of ready-made formulae. This might be due to the fact that static moments and centres are normally dealt with in depth in earlier classes on statics. This shows that it is hard to draw more general educational conclusions based on just one subject.

In the field of economics, Feudel (2016, 2017) investigated introductory textbooks regarding the use of the derivative. When using it for analysing cost functions he found that - contrary to the usual introduction in mathematics – the derivative is used as approximation for the difference quotient, and not the other way round. Moreover, since in the concept of “marginal cost” (cost for an additional unit) the denominator is 1, the latter is simply omitted which leads to inconsistencies regarding units (Feudel 2018). Since this is not in harmony with the concept image that is taught in mathematics Feudel recommends to enhance the latter in mathematics classes for economy students by emphasising this additional approximation aspect.

The above studies provide interesting insights into differences between usage of mathematics in theory development of just a few application subjects and mathematics education in application study courses but we are far away from having an overview comprising those subjects that draw more heavily on mathematics. More work on using mathematical reasoning, e.g. using definitions and arguing in theory development, is required.

Use of mathematical competencies when solving problems in application subjects

A further aspect of integrating mathematics education in application study courses is concerned with the mathematical competencies students need for working on assignments in application subjects. Biehler et al. (2015) and Kortemeyer & Biehler (2017a,b) define and use the concept of “Student-Expert-Solutions” (SES) for capturing the necessary competencies for solving tasks in a basic course on the foundations of electronics. Experts were asked how they expect students to solve existing tasks and these solutions were then analysed for necessary mathematical competencies using categorizations of mathematical problem solving as well as the modelling cycle comprising activities in mathematical modelling processes. Regarding a task related to an oscillating circuit, they found out that students have to understand a given circuit diagram that adheres to well-established conventions and set up corresponding differential equations. Moreover, they have to do “equation management” in order to solve for the unknown quantity. Validation can be done by comparing the resulting function with behaviour observed in a lab exercise earlier on. Similar results regarding statics tasks can be found in Alpers (2017a) where those parts of the modelling cycle are identified that are addressed in these tasks. Hochmuth & Schreiber (2016) investigate some tasks in signal analysis where arguments from signal theory can be applied to come quickly to a solution or a formal longer mathematical treatment is possible. Wawro et al. (2017) look at “meta-representational competencies” in quantum mechanics tasks where switching between representations and choosing an adequate one for a certain task is important.

Investigation of mathematical competences required at workplaces of graduates

Since service mathematics is not just directed at enabling students to understand and use mathematics in their application subjects but also in later work life, it is interesting to find out about the mathematical qualifications required at workplaces. This is a quite under-researched area when it comes to workplaces requiring an academic education and a sound mathematical basis. Alpers (2019) gives an overview of studies related to engineering workplaces. Such studies are based on observations of engineering work and/or on questionnaires and interviews. The more recent work by Firouzian et al. (2016), Quere (2017), van der Wal et al. (2017) and Engelbrecht et al. (2017) is based on analysing interviews with engineers. Firouzian et al. (2016) tried to identify the mathematical competencies (see Niss 2016) engineers think are important for their work. They conducted interviews with 20 Malaysian engineers from different areas and found out that modelling mathematically, problem handling and the use of aids and tools were mentioned most frequently. Quéré (2017) issued a questionnaire (237 responses) with engineers and interviewed 6 engineers in France in more depth. 53% of the responses stated that they “do not have a real need of maths” (p. 2236). Nevertheless, the answers regarding specific themes resulted in 69.4% mentioning “Scientific Computation”, 54% “Logic” and 49.6% “Modelling” as important for work. For analysing the interviews Quéré used the ATD framework and identified three “praxeologies” in engineering work where mathematics was involved: General types of tasks where reasoning, problem solving and elementary mathematical skills were used as “techniques” and justified; specific tasks associated with mathematical techniques where the techniques are just used and not really understood; a “mix of reasoning and using” (p. 2239) where the tasks required “the development of original techniques – almost a research work” (p. 2238). Van der Wal et al. (2017) investigate the “techno-mathematical literacies” (TmL) required in engineering work in order to “make an inventory of TmL in use” (p. 92). Since technological tools are used at most engineering workplaces mathematics is usually mediated by technology and the use is motivated by solving engineering tasks. They interviewed 14 engineers from different areas and identified TmLs which are organised in seven categories: data literacy; technical software skills; technical communication skills; sense of error; sense of number; technical creativity; technical drawing skills. It is not always clear what the specific mathematical part in the TmL is since they are intertwined with application knowledge, e.g. when a computation results in a car having a speed of 6000km/h then there must be an error (p. 97). Engelbrecht et al. (2017) interviewed 14 engineers and 9 lecturers (of application subjects) to answer the question whether in engineering work rather conceptual or procedural knowledge of mathematics is required. Conceptual knowledge comprises understanding and knowledge of connections between different pieces of information whereas procedural knowledge enables to perform step-by-step procedures for certain mathematical tasks. Among the interviewees, they identified two types of views, termed “conceptual view” and “balanced view”. Those having the conceptual view think that conceptual knowledge is much more important in engineering work (and should hence be more important in education); conceptual knowledge as stated by the interviewees is subdivided into interpretation (problem identification; application of mathematics) and understanding. In the “balanced view” bot procedural and conceptual knowledge are considered as important and must be combined. Conceptual knowledge arises from procedural one and one must also be able to go into the other direction, i.e. to come from conceptual knowledge to concrete procedures for solving problems. They conclude that in any case conceptual aspects should be more emphasised in mathematics education for engineers.

Curricular integration of Mathematics education into a study course

Based on an investigation of teaching materials, a survey and interviews with members of a science faculty, Barquero et al. (2011) found an epistemological attitude they termed “applicationism” where mathematics is seen as separate entity with its own systematic where only at the end applications might appear as add-on but the main responsibility of application and use lies with other specific subjects in a study course (if at all). Such an at most shallow integration of mathematics education into a study course does not foster the ability of students to make connections, see the role of theory development in application subjects and use mathematics for solving problems in context what Engelbrecht et al. (2017) called the “conceptual view” as outlined above. If the raison d’etre of service mathematics education is to serve the mathematical needs of students in study courses other than mathematics, this must be reflected in the mathematics curriculum. For this reason the SEFI Mathematics Working Group based its curriculum framework document (Alpers et al. 2013) on the concept of mathematical competence that emphasises the ability to understand, judge, do and use mathematics in relevant contexts which are provided by application study courses. So far, the identification of goals is essentially the result of practitioner discourses who have gained experience with application subjects and workplace usage of mathematics. A more systematic approach is reported in (Reid & Wilkes 2016) who brought together lecturers from mathematics and application subjects in order to set up “quantitative skills maps”. Those skills are defined roughly as the “ability to apply mathematical and statistical thinking and reasoning in context” but the lists that are provided are rather lists of mathematical topics (like “functions and equations”). Nonetheless, the lists helped in identifying mismatches between mathematics education and usage in application subjects. Faulkner et al. (2019) use a broad concept of “mathematical maturity” to “better understand what engineering faculty hope students learn from their mathematics course work” (p. 97). This construct comprises the concepts of “mathematical beliefs” after Schoenfeld, “mathematical competence” after Niss (2016) (see also Niss & Højgaard 2019), and “symbol sense” after Arcavi. They interviewed 22 engineering lecturers teaching classes which formally required some mathematics classes. Using thematic analysis they found that using and interpreting mathematical models as well as choosing and manipulating symbolic and graphical expressions were considered as particularly important. Moreover, engineering faculty stated that “computational tools reshape ‘what needs to be known’ to be mathematically mature” (p. 117). Regarding the modelling competency, mathematizing and interpreting are the most important parts. Engineering faculty thought that one of the major obstacles to developing this competence consists of the belief that mathematics has no relevance for engineering. The authors therefore recommend to include more modelling and context in mathematics education, to introduce and vary symbolic representations and to cut advanced analytic techniques in order to have time for this. Research on curricular integration of mathematics into economics and business study courses seems to be at a rather early stage. A study by Mills (2015) aimed at determining the calculus contents needed in such courses. Using a survey and interviews she found out that “differentiation, optimization, and rate of change” (p. 235) were stated most often. The interviews revealed where the concepts were used, e.g. to “minimize production cost”, to “do marginal analysis”, for “computing elasticity of demand” (p. 234). More work is required to get a deeper insight into how the concepts are used. This also holds for a study by Duran & Marshall (2019) on the mathematical “needs” of biological sciences students. Using journal content analysis, a survey and interviews with stakeholders involved in research and instruction in two fields of biology, they found out that the needs depend on the field where “Evolution and animal behaviour students need descriptive and inferential statistics, stochastic processes, differential equations, and mathematical models in general” whereas “developmental biology students … need only descriptive and inferential statistics, and mathematical models” (p. 807). The contexts for using mathematical concepts were mainly experimental design and an “overall understanding of mathematical models … as a minimum for scientific literacy” (p. 807). The authors suggest that there should be different offerings for students where a foundational course could address issues relevant for students intending to work in fields with less needs.

Brekke (2016) reports about a concept of embedding mathematics into a study course on electronics where a separate course was set up and – based on communication with lecturers from application subjects – close connections to examples from the latter are established. Kawazoe & Okamoto (2017) report on a concept for designing mathematics education in study courses on humanities and social sciences where the modelling needs of the study course lead to the introduction of mathematical concepts. They also emphasise "decontextualisation" of mathematics by letting students experience the occurrence of the same mathematical concept in different contexts such that they can recognise the common mathematical kernel.

If there is a larger basis of results from investigation of mathematics in application subjects and at workplaces (see the studies reported above) this should be used to inform curriculum development. Such research could also shed more light on the identification of a reasonable split of responsibility between the proper mathematics education and application subjects regarding the acquisition of mathematical competence. It is rather obvious that using mathematical concepts in theory building in application subjects as well as modelling and problem solving contributes to the development of mathematical competencies (cf. Gainsburg 2013). Engelbrecht et al. (2017) observed an "increased familiarity with the conceptual aspects of mathematics through its use in applied subjects throughout the education" (p. 574). Therefore, mathematics education might restrict itself to creating a kind of "introductory" familiarity the more detailed specification of which needs further research.

Scheduling mathematical topics in order to make them available in time as a result of setting up quantitative skills maps as outline above (Reid & Wilkes 2016) is not always possible when classes like fundamentals of electronics start right in the first semester. Hennig et al. (2014) report on a concept of introducing mathematical concepts as needed in early electronics classes, leaving a more thorough foundation to the treatment in mathematics classes later on. Another problem related to timing consists of rudimentary retention of mathematical concepts and procedures for later use in application subjects. Carr et al. (2013, 2017) investigated the effects of a scheme where students have to take mathematics tests in later years of study and need a high percentage (90%) for passing having several opportunities to repeat the test. This forces students to refresh their knowledge continuously. The tests have eventually (after several tries) a high pass rate but a test in the fourth year where it is no longer obligatory revealed that the knowledge had dropped significantly, so the tests have some effect but the authors recommend to "concentrate on conceptual understanding for longer and deeper understanding" (p. 148). Based on a case study on the retention of calculus concepts by a student of physical chemistry, Jukic Matic & Dahl (2014) found that the student, although being very successful at a previous test, had a very low retention. On the other hand, the student could give examples of calculus use in chemistry and successfully completed her bachelor degree. They conclude that "using calculus in other contexts does not in itself strengthen the 'original' calculus learnt in a more pure, formal mathematics course. Instead, it appears that these knowledge areas are disjoint" (p. 1181). In order to foster connections they recommend separate classes for different study courses where "tasks based on the movement from the symbolic world to the embodied one (in application concepts, B.A.), and vice-versa, should have a more prominent role in calculus courses designed for non-mathematics students" (p. 1183).

Making mathematics relevant for students

In a study on what motivates engineering students to study mathematics Harris et al. (2015) identified two "values" students see in studying mathematics: the "use value" and the "exchange value". Whereas the latter emphasises the additional esteem that is attached to a study course with harder mathematics requirements by employers, the former comprises the relevance of mathematics for understanding application subjects and solving application problems. Therefore, in order to motivate students to work hard on a subject like mathematics it is important to let them experience the relevance of mathematics to their study course and later professional career. This is also supported by Flegg et al. (2012): “We would argue that it is important to emphasise the relevance at every opportunity” (p. 728) although the students they questioned seemed to have already a very positive attitude towards the usefulness of the “skills”, “ways of thinking”, the “ideas” and even the “formal and rigorous aspects” of mathematics (p. 722). For the field of Humanities and Social Sciences, the study by Lopez Torres (2019) shows that students who had to take a course on “Topics in Contemporary Mathematics” were pleased by experiencing the course as easy but did not see any relevance for their further studies and work life perceiving the course as “enjoyable, but impractical and useless” (p. 1045). This leads to an attitude that makes later use of mathematics rather unlikely.

There are several ways to let students experience the relevance of mathematics to their subject of study. Providing students with respective resources like tasks or projects will be dealt with explicitly in the next sub-section. Recently, Goodchild et al. (2017) reported on a pilot study where modelling tasks were given to biology students which had a “positive effect on students’ attitude towards the subject” (p. 150). Goold (2015) had third semester students (environmental engineering) choose a mathematical topic and search for applications in their field of study, write a report and defend a poster. This resulted in an “increased awareness of the usefulness of mathematics in their future career” (p. 9) and motivated to work harder in mathematics education. Loch & Lamborn (2016) let 4th year engineering and multimedia students create animated videos that demonstrate to first-year students the relevance of mathematics. First-year students found that helpful but wanted to have the mathematics rather properly explained. The systematic investigation of this approach on the effects on recognizing the relevance of mathematics can be found in Dunn, Loch & Scott (2018). They used five animated videos produced by senior students in the first semester and found out using questionnaires that for many mathematical topics the estimation on their relevance went up, in particular for those topics the animated videos dealt with. In a direct question 84% of the students saw the relevance for their further studies and 75% for their future career.

3.2 Learning resources and arrangements: Analysing, initiating, and fostering students’ learning processes

The resources offered to students for fostering their learning processes have become much more varied in the last twenty years. Anastasakis et al. (2017a) promote a wider interpretation of the term “resource” including personal resources, written material like textbooks and students’ notes as well as digital offerings. They also recommend to classify resources rather according to usage (“actions”) and related goals than according to technical characteristics of tools (Anastasakis et al. 2017b). By issuing a questionnaire to second year engineering students (201 answers) they found that the most widely used resources were the course material (textbook in form of workbooks), the virtual learning environment, and students’ notes of lectures. The main goal when using these resources consisted of getting good marks on the examination which is well-known for engineering students. Similar results can be found in a large survey (662 answers) by Rønning (2014) regarding engineering students at the top technical university in Norway. There was a higher preference for lectures and textbooks and the preferred way of learning consisted of solving problems from books or previous exams in order to get good marks. This is also confirmed in the review of Biza et al. (2016) who summarize studies which observed a rather “shallow” search for patterns in examples in order to find something that can be slightly modified for solving a given task. They also found that explanations that would provide more meaning and justifications are often skipped. Moreover, students' beliefs and preferences seem to be strongly influenced by their discipline of study.

Resources for addressing relevance and ability to use in engineering

Different types of learning resources have been developed for reducing the gap between a purely mathematically oriented mathematics education and the usage of mathematical concepts and procedures to solve application problems. Wolf (2017) describes a "minimally invasive" approach where in the weekly mathematics assignment sheet for mechanical engineering students one out of four tasks was replaced by an application oriented task which relates mathematical concepts to an interesting application topic. He developed a concept for designing such tasks taking into account criteria like authenticity, relation to mathematics lecture, coverage of modelling aspects, being self-contained, constrained difficulty, and transferability. His quantitative investigation regarding motivation showed that students valued the tasks because they showed relevance and usage of mathematics but there are also many other motivational factors. If such assignments have no relevance for assessment then this can reduce motivation considerably.

A further resource to let students experience the relevance of mathematics to engineering are projects of different scale. Smaller obligatory application projects accompanying a regular lecture are described in (Alpers 2014), (Schmidt & Winsløw 2018), (Carr & Ni Fhloinn 2016), (Farrell & Carr 2018), and (Zetterqvist 2017) where the former two contributions address the first year of studies whereas the latter ones are concerned with special classes (DE and statistics, resp.) in years 2 and 3. In both former contributions ways to identify good projects are described. Schmidt & Winslow also provide a set of "didactical variables" which can serve to classify projects but can also provide guidance in setting up such projects ("authentic problems in Engineering") like breadth and depth of Mathematics 1 content, use of technology, breadth of the underlying engineering problem, use of modelling and work in models, acquisition of data.

The next step are courses where work on one or more projects forms the core of the course. Such courses might be voluntary additional offerings accompanying the "regular" mathematics education as described by (Härterich et al. 2012), (Rooch et al. 2016). (Rooch et al. 2016) state that there was no effect on self-confidence and a minor effect on motivation but the sample was rather small. (Goodchild et al. 2017) report on a cooperation between a mathematics education centre and a biology department where biology students had the opportunity to work on mathematical modelling tasks.

The project courses can also be obligatory courses as reported in (Alpers 2002) or normal elective courses for acquiring credit points (Wedelin & Adawi 2015; Wedelin 2016). (Alpers 2002) provides criteria for projects similar to those recommended by (Wolf 2017) and (Schmidt & Winsløw 2018). He also shows how project classes can be identified which facilitates the finding of ever "new" projects. Whereas in the concept of (Alpers 2002) each group works on one larger project in depth, the course design of (Wedelin & Adawi 2015) follows a different concept. Their course on "Mathematical modelling and problem solving", placed in the second year of engineering, requires students to work on around 30 smaller problems. The project work is surrounded by introductory and follow-up lectures. The criteria for good problems resemble several of the criteria for good assignments or projects mentioned above (e.g. "practical use", challenging to stimulate modelling skills, communication and problem solving, exemplary case for a larger class of problems). Using a variety of smaller problems and the course structure allows "students (to) get a chance to experience differences and similarities between problems, including higher-order patterns" (p. 422). Intensive feedback during project work (following the framework of "cognitive apprenticeship") and reflection on the whole problem solving process in the follow-up lectures support this.

Vasquez et al. (2016) report on an experiment where an application scenario (signal analysis) is not only used to let students experience the applicability of mathematical concepts but also to provide motivation for introducing new concepts like matrices and matrix inverses and to get a deeper understanding in the sense of APOS theory (action, process, object, schema).

The work reported above shows that meanwhile a lot of concepts are available on finding assignments for rich learning experiences relating mathematics to real usage in modelling and problem solving. Below we will report on investigations of usage of the resources by students.

Real usage of resources by students and development of competencies

In their survey of research on teaching and learning mathematics at the tertiary level, Biza et al. (2016) state that learning resources like textbooks are often not used as intended by their creators. For example, students rather perform shallow pattern matching looking for worked out examples that seem to be similar to the task at hand. Therefore, investigation of real usage of resources is important for estimating the actual learning effect. For some of the relevance related resources but also for other resources like digital ones, recent research on usage and the development of competencies is available.

Wedelin et al. (2015) investigate how students' modelling and problem solving skills developed during the special class offered by Wedelin (2016). In order to find out "how engineering students approach mathematical modelling problems, and how they develop the necessary skills and attitudes to effectively deal with such problems" (Wedelin et al., 2015, p. 559), they conducted a qualitative case study comprising interviews with students after two weeks and the analysis of final reports from 103 students containing overall reflections on their thinking about the important steps and challenges. The students recognised three main challenges: understanding the problem, exploring alternatives, and having the right attitude. Understanding the problem comprises problem solving strategies like reformulating the problem, splitting it into parts, visualisation and discussion with peers. Students stated that they often made quick decisions on which path to follow instead of looking for alternatives. As a strategy to avoid this they mention looking at the problem "from different angles" (p. 563), e.g. by changing representations. Regarding attitudes, students mention in their reflections to be open-minded, expect to try things out as well as the importance of patience and self-confidence. Using Schoenfeld's framework on mathematical thinking, the authors state that the main problems of the students related to meta-cognition and beliefs, not to missing mathematical knowledge: Students had problems to apply heuristic strategies and lacked "effective self-regulation skills" (p. 565). They were used to "solving clearly formulated problems in a linear way by following given paths" (p. 565) and their "beliefs can override self-regulation" (p. 565), e.g. when they think that there must be a known method for dealing with a problem. The authors interpret the contribution of the learning environment of the course in the framework of cognitive apprenticeship: students made their thinking visible by working on the tasks and teachers did the same by "showing not only the result of a solved problem but also how the solution was found, including alternative considerations, and by coaching students by providing feedback during the attempts to solve the problems" (p. 566). Moreover, the follow-up lectures served to reflect on the whole process. According to the students, this "deeply integrated supervision" was the most important factor in their learning process.

Viirman & Nardi (2017) investigate biology students' mathematical discourse during work on a voluntary unit on mathematical modelling using Sfard's theory of commognition. They recognised a development from the 2nd to the 3rd modelling session (out of 4): In the second session there was narrow guidance by the lecturer, and correspondingly the behaviour of students consisted rather of looking for "familiar rituals" than being open-minded and explorative. In the 3rd session with less scaffolding a more explorative attitude and behaviour emerged. They conclude that this reveals "the crucial role played by the teacher in facilitating the process" from ritualised to exploratory routines (p. 2280). But they also state that the stage of narrow scaffolding might "be a necessary step on the route to exploratory routine use" since "thoughtful imitation can be a transitory phase in transforming ritual into exploration" (p. 2280). Also relating to biology and life science students, Hall & Keene (2019) investigated how similar tasks placed within a kinematic context and within in a plant growth context much more relevant to the students were solved. They state that since the kinematic context was more familiar with the students they had only few problems with the respective task but they did not recognise the similar structure in the plant growth task. They conclude that contexts situated in the students' field of study should be used in calculus offerings for the corresponding study courses.

Gjesteland et al. (2018) investigate how mathematical modelling activities can be included in a traditional large class setting in order to activate students. They added a project task ("tracker task") where the students had to film an arbitrary motion, get numerical data by using tracker software, approximate the data mathematically, and make a poster explaining their reasoning. They measured the engagement of the students using the concept of "flow" (comprising phenomena like getting absorbed, feeling happiness, forgetting about other needs). Using a questionnaire including questions on flow, skills and challenges they recognised a higher flow with around 60% of the students, in particular when students estimated their skills and the challenges posed by the task on a higher level. They conclude that activation and a positive attitude can be achieved if the task is designed properly.

Liakos (2017) and Liakos & Rogovchenko (2018) investigate the development of mathematical competencies when biology students work on modelling tasks given to them in 8 complementary sessions added to the normal freshman course. The goal of these sessions was to activate students and to create competence profiles for individual students in order to follow their development. The study is based on the concept of mathematical competence as developed by Niss (2016). In order to measure the "strength of competence" activation (p. 216), they developed three dimensions: Task solving vision (understanding how the competency adds to the solution of the overall task); use of mathematical language/vocabulary; prompting (how much prompting was necessary). Moreover, they set up a scale ranging from beginning to exemplary with eight ratings. In Liakos et al. (2018) they used the results to identify 4 "key indicators" from the data and developed "learning types" for combinations of the values for these indicators. This work is still in the early stages but concepts for capturing the development of competencies are quite important in order to be able to evaluate the influence of interventions that are supposed to improve competencies.

During the last two decades, digital resources have permeated education in general and mathematics education in particular. The situation in mathematics is specific since symbolic and numeric computation and visualisations are available in form of calculators, separate programmes or Internet resources. A general overview of the affordances and risks for the area of engineering education including the potential for competence acquisition can be found in (Alpers et al. 2013, pp. 57-61). In the sequel, we present some recent results on the usage of such resources.

Hogstad et al. (2016) investigate the use of an educational simulation where velocity functions for cars can be specified and depicted and the motion is animated. The distance travelled is visualised as area under the velocity function. The environment hence offers symbolic, graphical and kinematic representations. Students were given open tasks where they had to construct velocity functions with certain properties relating to final velocity and distance. Beside the simulation, they used sketches of functions on paper and gestures. The tool was central since it provided the introductory setting and since its representations facilitated discussion and verification by providing visualisations and feedback. The sketches on paper served the discussion and elaboration of ideas inspired by the visualisation in the tool. Applying a commognitive approach (Sfard) to the same setting, Hogstad &

Viirman (2017) found that the tool was central to the discourse of the students. The environment "enabled the students to connect algebraic expressions, graphical representations and movement of objects" (p. 1247).

Kanwal (2018a) explores students' engagement with the digital resources provided in an online learning environment for calculus consisting of tutorial videos with notes (provided by lecturer), textbook, and the MyMathLab environment, an interactive environment for doing mathematics online. No traditional teaching is provided, homework and assignments are to be done in MyMathLab which also contains help in form of stepwise solutions of similar tasks and worked examples. All resources are based upon the textbook. In addition students have to work on a compulsory project where they have to provide a question bank on integration programming in STACK using Maxima. For analysing students work Kanwal applies a documentational approach (Gueudet & Trouche 2009) where she is looking for paper and pencil based techniques and instrumented techniques used by the students. The case study with three students revealed that the students preferred to work in MyMathLab because it provides quick and relevant help. This is seen as a "pragmatic use". Videos were watched when they were needed for getting a better understanding. Kanwal also observed an increasing use of additional digital tools like Wolfram Alpha where students applied "more instrumented techniques instead of paper and pencil techniques" (p. 152). She concludes that the "preference for more pragmatic instrumental techniques" is likely to be caused by "online final examinations where they could use the resources" (p. 153). In Kanwal (2018b), she uses the framework of mathematical competencies to analyse the students' development when working in the same environment. She observes that when students get seemingly different results from online resources and the textbook they think about different representations and handle symbols in order to check for equality. She concludes that "representing mathematical entities, handling mathematical symbols and formalism, communicating in, with and about mathematics, and making use of aids and tools are *in action*" (p. 1296).

Zetterqvist (2017) describes a concept for a course on probability and statistical inference for several engineering study courses where the use of digital resources is fully integrated and adheres to the concept of constructive alignment (Biggs & Tang 2011). Before the intervention the course suffered from low motivation and grades and a missing sense of applicability. The innovative concept included the use of applied problems ("mini projects") tailored to the engineering course at hand which had to be solved using Matlab which was also used for visualisation and exploration of statistical concepts. Moreover, in order to ensure the availability of probabilistic concepts at the end of the first part of the course, an online test was administered. Zetterqvist describes different stages of introducing the innovations and provides data on student scores in examinations and student satisfaction stated in course evaluations. She observed that the better the innovations were aligned with and explicitly connected to the goals of the class, the study course and the assessment regime, the higher was the gain in examination results and acceptance. The simple provision of a technology-based task with real data is not sufficient. She concludes that "students' learning could be supported through the use of digital resources if they are aligned in the course and seem valuable for students" (p. 111). Problems and data sets must be "included in tasks which students find reasonable and beneficial in some way" (p. 111).

Fredriksen et al. (2017) explore a flipped classroom setting in order to identify sources of "tension". Students were assumed to watch a video in advance and then work in groups on tasks during lecture time. The authors used a questionnaire which particularly asked for such tensions (sources for inconvenience). They found out that students felt uncomfortable with not being able to work alone on the tasks first and with larger differences in understanding within the group. They were also concerned with the tasks not being relevant to the final examination. Students expected to be taught by the teacher as opposed to the "rules" of flipped classroom stating that students had to watch videos first.

On the other hand, they found out that students who were well prepared "seemed eager to learn more about the concepts behind the procedural mathematics" (p. 2063) indicating the potential the flipped classroom concept might have. Tague & Czocher (2016) address the problem that students did not feel well-prepared for in-class activities by materials offered for out-of-class that should be studied before. They perceive this as a problem of incoherence where they distinguish between "*instructional incoherence* (cohesion and coordination among instructional materials) and *curricular coherence* (the extent to which mathematics content is logically, cognitively, and epistemologically sequenced)" (p. 224). They developed a concept for identifying cognitive obstacles to and mathematical abilities needed for working successfully on in-class problems and developed out-of-class materials that addresses these issues and which, according to a survey, let students experience the latter as appropriately connected to the former. The recent research shows that a simple model of flipping without a proper concept of orchestration of materials and activities and without taking into account student attitudes and behaviour (cf. Triantafyllou et al. 2015) might turn out to be rather counterproductive. In a study on engineering students' engagement with recorded lecture videos (RLV) that were offered in addition to face-to-face-lectures in Australia and the UK, Trenholm et al. (2019) found out that "regular RLV use may be adversely affecting student academic performance because they enable students to adopt more surface approaches to learning mathematics" (p. 17). Beyond mere usage statistics, the type of engagement and corresponding learning processes must be taken into account.

3.3 Transition issues

Transition from secondary to tertiary education has been discussed for decades (Artigue et al. 2007; Alpers et al. 2013). Whereas in mathematical study courses the transition from more informal school mathematics to strictly formal and abstract university mathematics based on definitions, theorems and proofs constitutes the major problem on the cognitive level, in service mathematics the transition problem is often perceived as one of missing prerequisites from school education when the entrance to university education broadened and the student population became much more heterogeneous. The study by Nortvedt & Siqveland (2019) based on data from Norway shows considerable deficiencies regarding lower secondary mathematics education like arithmetic and basic algebraic manipulation with mathematics and engineering students. They assume that such missing fluency with basic concepts forms a major obstacle to understanding and working with more complex concepts.

In order to provide clear orientation for schools and politics, it is necessary to turn complaints about deficiencies into more specific expectations what mathematical knowledge and competencies are required for certain study courses. In a large investigation Neumann et al. (2017) performed a Delphi study asking lecturers at German universities (both traditional and universities for applied studies) what they perceive as necessary mathematical capabilities when entering a MINT study course (Mathematics, Informatics, Natural Sciences, Technology). With 952 answers the study has a very sound data base and it revealed a high level of agreement. Lecturers expect that those topics and procedures that are formulated as goals in official documents are known and well trained which refers particularly to grades 5 to 10 (secondary level 1). For themes dealt with at secondary level 2 (grades 11-13) they are content with an intuitive understanding. There was some disagreement on the necessity of more formal abstract representations and proof but since MINT comprises mathematics as well as engineering this is not surprising. Lecturers also expect aspects of mathematical competencies like problem solving or dealing with tools. Moreover, features of personality like persistence and curiosity were also included in the set of prerequisites.

Although studies like the one described above can give valuable information for future improvements, universities nonetheless have to deal with the current situation where students have difficulties when entering university to cope with what is expected resulting in high drop-out rates. Therefore, during the last three decades several offerings have been installed ranging from additional tutorials and buddy programmes to bridging courses and mathematical support centres (cf. Alpers et al. 2013, pp. 53-56, for engineering; Bausch et al. 2013). Engelbrecht and Harding (2015) provide a framework for such first year interventions. They identified five challenges for helping first year students: mathematical preparedness; non-cognitive aspects (new environment); continuous and independent learning; readily available support; repeated exposure/multiple opportunities for achieving fluency. For each challenge they provide corresponding interventions that were installed at the University of Pretoria and for some there are quantitative results on effectiveness. These comprise bridging courses and an adaptation of the curriculum in order to decrease the pace, identification of “at-risk students” with special counselling and support, digital tools (online homework, quizzes) for continuous engagement, drop-in service with assistants, and overview lectures and summer/winter schools with intensive training for repeaters. This concerted approach led to an increase in pass rates from 71% to 89%.

With such an abundance of support offerings available, a categorisation of such offerings is needed. Liebendörfer et al. (2017) provide a framework for goal dimensions which is necessary for being able to evaluate interventions against their goals. They considered 44 “Projects of Mathematical Learning Support” (PMLS) at German universities that try to help students to “acclimatise to university mathematics” (p. 2177). Four types were identified: bridging courses, mathematics support centres, redesigned lectures, and measures that run in parallel to courses (online material, special tutorials). By performing document analysis with inductive category formation and by clarifying goals in interviews they identified two clusters of goals: educational goals and system-related goals. The first ones are concerned with what each individual student is supposed to achieve whereas the latter refer to whole systems/institutions. Among the educational goals there are knowledge goals (e.g. refreshing school mathematics), action-oriented goals (mathematical modes of operation, learning strategies), attitudinal goals (beliefs, perception of relevance) and learning and working conduct. The system-related goals include issues like creation of prerequisites with the majority of students, improvement of formal study success (drop-out rate, passing rate), improvement of teaching and feedback quality, promotion of social contacts, increasing transparency and supporting specific student groups. The system-related goals serve three kinds of purposes: “preservation of the institution”, “improve the environment for students’ self-directed learning”, and “creating equal study conditions” (p. 2183). This taxonomy can be used by practitioners to clarify their goals and teaching practice.

Biehler et al. (2018) used the taxonomy for performing a comparative evaluation of six bridging courses at five German universities that were aimed at students of different study courses (mathematics, engineering, physics, teacher education, computer science). In the study, students were given a questionnaire after the course that was based on the goal categorisation described above. The students should state to which extent they thought they had achieved the goals. This “empirical profile” was then compared to the “theoretical profile” that was intended by the course provider. As a result, the authors found no big difference for courses that were directed at mathematics or mathematics education study courses where goals included an introduction to more formal mathematical thinking to be encountered in the respective study course. For other courses there were sometimes narrower views to be found with students (just repetition of school mathematics) but sometimes the students’ views were even broader. Such comparisons can provide valuable feedback for course designers and lecturers.

Other studies measure the success of bridging (or “preparatory”) courses using quantitative data based on test and first year examination results. There are mixed results ranging from minor to significant effects (see the discussion in Greefrath et al. 2017, p.164). In a large study by Greefrath et al. (2017) regarding beginners of study courses in electrical engineering and computer science, the authors tried to relate data on prior achievements at school and participation in two types of preparatory courses (on-campus and e-learning) to the results of an obligatory mathematics test in the first week and examination results in calculus and linear algebra at the end of the first semester. They found a large difference in the pass rates of the mathematics test depending on the participation in the preparatory course but only a minor effect regarding to the average achievement; the influence of prior school education (attendance of a course in advanced mathematics) was much more influential than participation in the preparatory course. This demonstrates quite well that when effects are judged the result depends very much on the criterion applied and hence on the goals of the intervention. The authors also identified a significant correlation between the mathematics test results and the final exams. They conclude that the test which measured predominantly school knowledge related to handling “symbolic, formal and technical elements in mathematics” (p. 149), i.e. carrying out operations and procedures, turned out to be a good predictor of academic success. Participation in the preparatory course did not have a significant effect on the final examination. Therefore, the authors see a preparatory course as an “initial step in student support at universities” (p. 164). Derr et al. (2018) investigate the effect of an e-learning “pre-course” for students of different kinds of engineering study courses. They had data on prior knowledge (final school examination), pre- and post tests regarding the pre-course, and final examination in “Mathematics I” as well as survey data on affective variables, learning strategies and engagement. They confirmed that prior knowledge (final school exam and pre-test) had the “strongest impact” on the final examination explaining 21% of variation. They also recognised only a minor effect regarding the gain (difference of pre- and post test: 5.5%) where the gain score was higher (9.1%) when students additionally took part in an e-tutoring course and a weeklong face-to-face class. Engagement measured by test attempts is significantly related to gains showing that not only participation in an offering but also motivation and efforts are important. The gain score was also significant for the Mathematics I result but ”compared to the dominant role of prior knowledge, the effect was not very strong” (p. 920). Rylands & Shearman (2018) investigate the effect of additional workshops provided in a foundation class. Depending on the results of a mathematics placement test, students had to take this class first before proceeding to the regular first mathematics module. As opposed to other data reported in the literature, they stated a relatively high attendance rate (57.9% attended at least one workshop) which they attribute to the placement test making students look for help. They also found that students belonging to the “high support group” participating in at least half of the workshops performed significantly better in the final exam but they also state that there is a correlation between making use of support offerings and engagement in regular offerings like tutorials because of which one cannot simply attribute better performance to high support. In the next paragraph, we will deal with the effect of learning behaviour more generally.

In order to address particularly those students with adequate support measures which are at risk of failing, the identification of valid performance predictors is necessary. Hieb et al. (2015) investigate factors that predict grades in the Engineering Analysis I exam taking into account ACT (American College Testing) maths scores, scores in an algebra readiness exam (ARE) taken before and after a summer school intervention before the first semester as well as learning strategies and motivational factors. Using a multiple linear regression model they found that ACT scores, ARE scores after the summer school as well as time and study management, goal orientation and test anxiety have significance in predicting Engineering Analysis grades explaining 42% of the variation where ACT explains 23.9% and ARE explains 5.1%. They conclude that beside interventions improving algebraic competence also help in learning strategies and management should be considered. In a larger study on the determinants of beginning business and economics students' basic mathematical skills, Laging & Vosskamp (2017) similarly found that "type of school graduation, final school grades, pre-knowledge and self-beliefs are essential determinants" (p. 108), and "regular learning" is considered as a particularly important component of learning behaviour. Gradwohl (2018) conducted a study with 182 engineering students at the beginning of the 2nd semester where she explored motivational aspects (goal orientation, interest, expectancy and incentive value) and learning strategies and tried to relate them to study success measured as achievement at the first semester final examination. The results of a corresponding questionnaire show that "final school grades, maths self-beliefs, value of maths, the type of matriculation and the choice of advanced courses in school are meaningful predictors of performance in engineering maths" (p. 131). As a consequence, she suggests that support measures should also address the development of students' self-beliefs. Griese & Kallweit (2017) try to relate study behaviour in first year engineering maths to success in examinations. They performed a survey containing questions on learning behaviour in weekly assignments, lectures, and tutorials as well as on deep and surface learning and effort. The questionnaire was issued three weeks before the end of the first semester leading to 458 data sets from which 202 could be used to match with exam results. Regarding the learning behaviour, the highest score was observed for attending lectures which might be due to a novel approach using a script with gaps available before lectures. Cluster analysis led to two clusters for learning behaviour: sensible, continuous and diligent learning behaviour on the one hand and superficial and irregular learning on the other hand. As dominant factors for success Griese & Kallweit identified work on weekly assignments, attending tutorials and avoidance of surface learning.

Bergsten & Jablonka (2017) explored the differences between acting in a school environment and in university study courses on mathematics in engineering. More specifically, they focused on students' perceptions of "differences in teaching and classroom organisation", "change in expected learning habits and study organisation", "differences in atmosphere and sense of belonging", and "differences in pedagogic awareness of teachers" (p. 454). They conducted interviews with 60 engineering students at two Swedish universities at the end of the first year asking explicitly for university experiences compared to high school ones. For analysis they used a thematic approach and interpreted the data within a framework of student identity. The students experienced changes from a mainly individual study approach to a more collective one working with peers after lectures. They found mathematics harder than other subjects and more time consuming and demanding than expected. Lecturers are seen as knowledgeable and helpful even when contact is less personal. In terms of identity, Bergsten & Jablonka state that at university students are "apprenticed into the principles of the discourse" whereas at high school "they remain in a dependent unauthorised position" (p. 459). Students "experience that they are expected to exhibit an increased degree of autonomy" and a "shift from an individualised to a collective approach, including informal group studies on their own initiative" (p. 459/460).

3.4 Assessment

In the outline of research results regarding special interventions in previous sections, at several places the importance of assessment was mentioned. Students take goals and settings more serious if they are aligned with assessment schemes as promoted in the concept of constructive alignment (Biggs & Tang 2011; see also Alpers et al. 2013, pp. 68-73). If, for example, course goals address conceptual knowledge and mathematical competencies and the examination only requires the knowledge of computational procedures then students concentrate their efforts rather on mastering the latter.

Assessment is usually subdivided into two forms, formative and summative assessment, where the former serves the purpose of providing continuous feedback on the state of the learning process to learners and lecturers whereas the latter informs about the final achievements of the process in form of grades. Therefore, for supporting learning processes formative assessment is very important. Ni Fhloinn & Carr (2017) provide an overview of different forms of this kind for mathematics education in engineering study courses including in-class exercises, marked homework, and quizzes on paper or electronically administered using online assessment or electronic voting systems. They emphasise that including such ways of formative assessment sends “a message to students about our own perception of its importance and the benefits can be accrued by both staff and students as a result” (p. 466). Particularly online quizzes have been the topic of recent research on computer-aided assessment. Greenhow (2015) states some requirements that should be fulfilled for effective use like the support of random parameters to create classes of exercises and the possibility to encode algorithms for correct and incorrect answers. Lowe (2015) uses online quizzes in different forms in a distance learning environment. His concept consists of 14 formative practical quizzes with unlimited attempts and no contribution to grades (one per week), 5 “interactive computer-marked assessments” (iCMA) with 1 attempt, and 4 written assignments. The iCMAs provide 15% of the common grade for iCMAs and written assignments. Drawing from usage data provided by the system, Lowe observed that the number of attempts for practical quizzes which should serve as preparation for the iCMAs decreased very quickly and was “disappointingly low” (p. 148) overall but “those students who use them appreciate the facility to practice their mathematics and self-assess their skills” (p. 148). Gaspar Martins (2018) investigates whether the goal to make students work more and continuously and to increase their awareness of their understanding was achieved by quizzes used in two engineering maths courses with about 100 students each. Students got one quiz per week (14 overall) and the results of the best 10 resp. 12 quizzes provided up to 10% of the final grade but only if the students achieved at least 45% in the traditional final paper and pencil examination. She observed a “high rate of attendance” (p. 109) and concludes that the goal stated above was achieved. There were also better results in the final examination compared to previous years but since there were more changes made this improvement cannot be clearly attributed to the quizzes. Rønning (2017) explored how Maple TA® tests in calculus I and II classes for engineering students influenced their learning behaviour. There was one test per week, 6 tests needed to be passed for admission to the final examination, and with more than 10 tests passed the results provided 10% credit for the final grade. Additionally, four problem sets had to be solved as written hand-ins and were graded and commented on by teaching assistants. The tests consisted of multiple choice questions or required numerical or symbolic input. Students had an unlimited number of attempts. Rønning got data from a survey administered to the students with a response rate of 40-50%. Students were content with the quick feedback the system provides but they complained about the quality of feedback (right/wrong). Still, more than 50% admitted that they still copy from others where the deadline and the requirement for admission might contribute to this behaviour. In additional interviews with seven students Rønning detected several types of influence on student behaviour. Students were “hunting for the answer”, i.e. they work less carefully since only the final result is judged by the system. They stated that they worked more diligently on the written hand-ins: “It is more important when a person looks at the solution” (p. 101). They learned more from written hand-ins because they had to include more explanations and were getting feedback. Students were also frustrated at times when the system just gave them “wrong” as an answer and they could not figure out the reason. Although additional exercises were provided in order to prepare for the Maple TA tests, students rather went directly to the test which were essential for admission. The results by Lowe, Gaspar Martins and Rønning show again that relevance for and contribution to the final summative grade seems to be the main factor influencing student behaviour particularly in service mathematics.

Summative evaluation in service mathematics still seems to be dominated by written examinations (see Alpers et al. 2013 for engineering) although the introduction of different kinds of projects as described in section 3.2 also allows for the assessment of competencies demonstrated in project group work and in project presentations particularly in smaller classes. As Altieri & Prediger (2017) state regarding the situation in Germany, procedural knowledge comprising algorithms and procedures for solving mathematical tasks still plays a major role and students fail because of problems with applying this knowledge. In order to get a better understanding of the problems and to be able to address them, they analysed student errors in examinations leading them to differentiate between procedural "knowledge-in-mind" (knowing which procedure to use and how to perform it) and "knowledge-in-use" (ability to perform including aspects of accuracy, speed, flexibility). In a study with about 1200 engineering students in the first (Test 1) and then the second semester (Test 2) they had students solve procedural tasks under time restrictions and related errors to missing "knowledge-in-mind" resp. "knowledge-in-use" for different performance groups. They found different developments for the performance groups which also depended on the mathematical topic. As a practical consequence they recommend to strengthen the "knowledge-in-use" part by fostering "automatization and flexibility" whereas "knowledge-in-mind" "might develop effectively by a permanent application of basic procedures in complex situations … with links to conceptual knowledge, metaprocedures and the web of procedures they are embedded in" (p. 145).

Regarding the assessment of competencies, the mathematical modelling competency has been thoroughly treated within the ICTMA community (www.ictma.net), see for example (Blum et al. 2007, pp. 403ff). Recently, Liakos & Rogovchenko (2018) and Liakos et al. (2018) developed a more refined model of capturing progress in competence acquisition that was already described in section 3.2. But there still seems to be a long way from using such a scheme in research settings to performing summative evaluation in study courses.

If it is the goal of mathematical education to enable students to create algorithms for solving mathematical problems, alignment of goals and assessment requires similar tasks in examinations. Kruse & Seifert (2018) present a concept for a course on numerical analysis for engineering students where Matlab® was not only used in tasks and small projects in normal exercise sessions but also in the final examination where short algorithms using realistic data had to be developed. They give an overview of legal, organisational, technical and educational aspects that have to be taken into account and provide a corresponding check list. An evaluation by students revealed that about 64% felt well-prepared and nearly 98% "were convinced that the practical implementation of programming matched the course" (p. 36).

Even when students successfully pass an examination, there still remains the question of how long they will retain their knowledge. As already stated above, Engelbrecht & Harding (2015) recommend repeated exposure and multiple opportunities for securing a long lasting success of learning, and Carr et al. (2013) address the problem of continuing deficiencies by inserting an "Advanced Mathematics Diagnostic Test" at the beginning of the 3rd year. But still retention and its relationship with characteristics of the teaching and learning process (e.g. fostering conceptual knowledge, establishing connections between concepts and applications) are to be investigated more thoroughly.

3.5 Teachers and teaching practices

In service mathematics teaching, the teacher community is much more heterogeneous than in teaching mathematics in mathematics study courses. Already in the discussion document of the first ICMI study (Howsen et al. 1988, p. 9) the question of who should teach mathematics as service subject was posed. The recent study by Vosskamp (2017) capturing the situation in study courses in business administration and economy showed that mathematics modules were offered by mathematics departments (60%) as well as by economics departments (37%). When organised by mathematics departments nearly all lecturers had a degree in mathematics whereas in the second case this applied to only 50% and 33% had a degree in economics or business administration. Vosskamp observed that "for external education (i.e. by mathematics departments, B.A.) … the extent of education was higher, mathematics educators had a better professional position, lecturers had a stronger academic background in mathematics, curricula were more homogeneous and textbook selection was broader" (p. 186). Based on a study on factors influencing students' attitude towards mathematics which will be discussed in more detail below, Mesa et al. (2015) recommend assigning full-time faculty for calculus courses in order make the high availability and engagement more likely that are particularly appreciated by students. Wolf (2017, p. 478) emphasises the importance of personal continuity for facilitating sustainability of interventions like the introduction of application-oriented tasks described in section 3.2.

Bingolbali & Ozmantar (2009) investigate the factors influencing mathematics lecturers' service teaching. Based on interviews with 22 mathematics lecturers in Turkey who taught both in mathematics courses and in application study courses, they found that "whilst these lecturers teach the same course with similar content in departments other than their own, they emphasize particular aspects and examples, set different types of examination questions and make use of different textbooks and materials" (p. 602). In engineering courses they were more application oriented and less theoretical and avoided proofs in examinations. Their reasons for doing so were categorised by the authors as follows: "student needs" (86%), "student expectations" (32%), "departmental features" (59%), "departmental demands (45%), "curricula and teaching hours" (59%). They conclude that "the lecturers' interpretation and perception of the characteristics of departments is the leading factor in their approaches to teaching the same topic in different ways and with different emphasis" (p. 613). The study by Hernandez-Gomez & Gonzales-Martin (2016) investigates how the background of teachers in engineering courses influences their views and actual teaching practices when teaching calculus. Using ATD as a theoretical framework, they try to relate the different praxeologies lecturers experienced in their former education and work to their opinions and practices. In a small interview-based study with six lecturers having (sometimes mixed) backgrounds in mathematics, engineering, and mathematics education, they found out that a lecturer with a background in mathematics (Bachelor) as well as engineering (Master and PhD) had a stronger conceptual approach and at the same time saw the need for real applications and was aware of use in later application subjects. A lecturer with just an engineering degree (Bachelor) and Master and PhD degree in Mathematics Education was much more restricted to teaching procedures according to a book. The authors suggest that a lack of teacher training might contribute to a high influence of formerly experienced praxeologies. The study indicates that the academic and professional background is likely to play a role in teaching visions and practices but it is clear that considerably more research is needed to get a better picture.

Treffert-Thomas et al. (2017) focus on the views and practices related to the special theme of mathematical modelling that is often considered as being vital particularly in service mathematics (see e.g. the mathematical modelling competency in Niss, 2016, and Alpers et al., 2013). They explored Norwegian lecturers' views on the aims of Mathematical Modelling (MM for short) in professional practice and in teaching, the actual use of MM and reasons for non-use as well as preferences of and obstacles to using MM. About 120 responses from an online questionnaire sent to about 500 lecturers that used Likert scale and ranking items as well as open boxes were analysed using the categorization of aims of MM developed by Kaiser and Sriraman that distinguishes between a "realistic", "epistemological", "socio-critical", "contextual", and "educational" perspective. They found that "the realistic perspective on the aim of both modelling and the teaching of modelling was a prominent one" (p. 132) where realistic means that MM and its teaching serve to solve authentic problems and to develop the ability to do so. Regarding the use of MM in their teaching, 74% of the respondents "indicate that they used models or modelling in their teaching" (p. 133) in order to "illustrate theory" (30%), to "motivate new theory" (16%), in a separate modelling course (16%) or as "applications after small pure units" (9%). The authors state that those who used MM in their research were significantly more likely to use it in teaching. Reasons for not using MM were categorised as follows: "nature of mathematics", "institutional conditions", "lack of teaching skills", "student profile" where about 50% referred to the first category. They thought that modelling was not relevant or appropriate e.g. when teaching pure mathematics. The authors recommend to make MM more explicit in mathematics curricula and to encourage collaboration between educational researchers and lecturers to raise the awareness of aims of MM which is in line with research on pedagogical awareness in general (Nardi et al. 2005).

Treffert-Thomas & Jaworski (2015) organise the literature on teaching practice into three categories: "*professional literature*, in which we gain insights into the ways in which other teachers/lecturers have thought about their teaching and the approaches/frameworks they have used; *research literature* which offers *what is known*, findings from research that can enable more informed approaches to teaching; and *pedagogical literature* that deals overtly with developing and enhancing teaching, through the lecturer engaging with new ideas (for example those offered in the professional literature), attending to research findings or using specific tactics or teaching approaches recommended by the authors" (p. 259). Speer et al. (2010) noticed that teaching practice in undergraduate mathematics is strongly under-researched with just a few studies available. They provide a framework for analysing teaching practice using the following "dimensions": "allocating time" (e.g. for examples or additional explanations); "selecting and sequencing content"; "motivating specific content"; "asking questions, using wait time, and reacting to student responses"; "representing mathematical concepts and relationships"; "evaluating completed teaching and preparing for the next lesson"; "designing assignments and evaluating students work" (pp. 107-111). These issues frequently showed up in some genuine research studies reported below.

Treffert-Thomas (2015) investigated in depth the teaching practice of one lecturer of a course in Linear Algebra. Although the course was given to mathematics students, not as service mathematics, it sheds light on how teaching practices can be explored. The author analyses observations of lectures and subsequent discussions with the lecturer using grounded analyses where she structures her themes according to activity theory after Leontiev using the dimensions of actions and underlying goals. She identified a hierarchical structure leading – in the dimension of goals – from "Engagement with mathematics", "Intuitive understanding", "Acquisition of mathematical language", and "Conceptual understanding" to "Mathematical competence". Corresponding actions include the presentation of examples first in order to foster engagement, the work with R^n as vector space to get an intuitive understanding, and the verbalisation of intentions on different levels to make students conscious of the purpose of concepts and theory. The model could also be applied to other fields of mathematics teaching including service teaching. It serves to raise awareness of different levels of potential goals and corresponding actions.

Viirman (2015) uses Sfard's theory of commognition in order to investigate the discursive practices of mathematics teaching by seven lecturers in Sweden who taught the function concept in basic courses mainly for engineering students. Among the characteristics of discourses he was particularly interested in repetitive patterns, so-called "routines". He identified three types of routines: explanation, motivation, and question posing. Explanation routines were sub-divided into "reference to known mathematics facts", "summary and repetition", "different representations", "reformulation in everyday language", and "concretization and metaphor" (p. 1171/72). Motivation routines had two purposes, motivating mathematical activities and raising students' interest. These routines have as sub-types "utility" (useful for later purposes), "nature of mathematics" (e.g. being precise), "humour", "result focus" (good for examination). Question posing was used "very differently, with regard to both purpose and frequency" (p. 1175). Simple "control questions" (any problems?), "facts" (used for activation), "enquiries" (for reflection on mathematics), and rhetorical questions that demonstrate and explain the internal thinking of the lecturer. Rhetorical questions were by far the ones observed most often. The results show that the lens of commognition allows to detect patterns in the discourse of mathematics teaching. It would be interesting to see whether the discourses are in some way specific to service mathematics and whether relationships can be identified between the types and mixture of discourses and student learning.

Van der Wal et al. (2019) investigate teaching strategies for fostering techno-mathematical literacies. In their design research study, they set up an applied mathematics course for life science students where the students had to work on three "cases" related to their field of study. After working on their own, there was a feedback session where groups presented their results and the lecturer applied a questioning approach based on inquiry-based learning. An analysis of this approach revealed that the lecturer fostered the acquisition of techno-mathematical literacies by several strategies including asking "students deeper questions about data, tables, formulas, and figures", asking "students to elaborate on the answer", and letting "students discover their mistake by stimulating thinking about the logical answer". They conclude that the "feedback hours, with their classroom discussions, and usage of IBL questions, seem to contribute to the learning of TmL". (v.d. Wal et al. 2019, no page numbers).

In a synopsis of four studies on teaching in lecture rooms and in small groups, Jaworski et al. (2017) show how on the one hand existing theoretical constructs can be used for analysing observations of teaching practice and interviews with lecturers, and how on the other hand new theory can be developed and existing theory can be refined using grounded analysis (see the study by Treffert-Thomas, 2015, which was one of the four studies). The studies investigate how teachers conceptualise teaching and how teachers use resources to help students in the process of mathematical sense-making and developing mathematical meanings. They use constructs from Activity Theory after Leontiev, the Teaching Triad (TT) after Jaworski and the Spectrum of Pedagogical Awareness (SPA) after Nardi, Jaworski & Hegedus to categorise observed teaching and to make sense of teaching episodes. The SPA allows to position teaching practice within a spectrum reaching from "naïve and dismissive" via "intuitive and questioning" and "reflective and analytic" to "confident and articulate" (Nardi et al. 2005). The TT facilitates the interpretation of teaching episodes by characterising them as acts of "Management of learning", demonstrating "Sensitivity to students" or defining "Mathematical challenges" for students. These constructs enabled the authors to investigate the lecture setting much more thoroughly than just viewing it as a "transmissionist" form of teaching. As also observed in the analysis by Viirman (2015) reported above, teachers used questioning during the lecture and responded to students trying to incorporate their ideas thus showing student sensitivity. They also adjusted their level of difficulty depending on students' answers thus providing adequate mathematical challenges to their students. The authors conclude that "there are alternative ways for teaching in the instructional context of the lecture and identification of these might be used as a lecturer's 'self-awareness springboard' towards improving teaching" (p. 183). In one of the small group teaching settings, a tutor should help in work on weekly problem sets accompanying a lecture, but the tutor had to discuss basic meanings first because they were not understood. Here, TT also served well to interpret the questioning approach as management of learning, taking into account the responses as act of student sensitivity and developing corresponding mathematical tasks as challenges. Some of the students were concerned with not working on the weekly problems sufficiently, so there was a mismatch between intentions of the tutor ("grasping basic meaning first") and expectations of the students. A similar situation of "tension" was observed by Jaworski et al. (2012) (see also Jaworski & Matthews, 2001, and Jaworski (2013)) in the ESUM project which sought to improve the sense-making process of engineering students by including elements of inquiry based teaching for investigating functions using GeoGebra. Although examination results were better than before the intervention, many students were "strategically focused towards achieving the best outcomes in the official assessment and engaging in familiar practices that they perceive as helpful, albeit at the expense of a deeper understanding" (p. 148). The authors interpret the situation using categories from Activity Theory juxtaposing different motives of teachers and of students for their activities where teachers seek to enhance understanding and to enable students to make use of concepts and students try to achieve good examination results in an effective and efficient way.

From this research it is clear that identifying factors that influence students' attitudes towards mathematics is important. This was intended by the large national study undertaken by the MAA in the US regarding College Calculus (Bressoud et al. 2015). They conceptualised student attitude as a composite of "mathematical confidence", "enjoyment" and "persistence", and investigated the influencing factors by a large student survey covering several thousands of students. It turned out that teachers and teaching practice ("Good Teaching") had a major influence on the attitudes. "Classroom interactions that acknowledge students", "Encouraging and Available Faculty", and "Fair Assessment" (p. 84) were identified as the three main constituents of "Good Teaching". Mesa et al. (2015) investigated these constituents in more detail by making "in-depth site visits" at selected institutions, interviews with student groups and classroom observations. They summarise their results as follows: "Good teaching involves, first, a teacher who is encouraging of students' efforts to learn and who is available to answer their questions and to support them in their learning; second, a class environment that fosters interactions geared at eliciting students' participation as a means to promote mastery and understanding of the material; and third, tests and assignments that are perceived as fair" (p. 89): The most popular interaction they found in their classroom observations was questioning by the lecturer, response by students, evaluation/feedback by lecturers.

Studies on teaching practices and resulting student attitudes could also be used for teacher professional development in service mathematics. There is hardly any literature on concepts for such development related to mathematics teaching at the tertiary level. Treffert-Thomas & Jaworski (2015) intend to contribute to such development by making teachers aware of "what can we learn from literature" summarising results that can improve pedagogical awareness and practice (see Nardi et al. 2005). They consider cooperation between teachers and mathematics education researchers as particularly fruitful. (Treffert-Thomas 2015), (Wedelin & Adawi 2015), (Wedelin et al. 2015), (Viirman 2018) provide successful examples of such cooperation.

Wood et al. (2011) investigated the Australian situation realising that often the only preparation for teaching was a more general induction course for new teaching staff. They investigated the needs as perceived by faculty in a survey and found the following areas for further training: "presentation skills"; "facilitating group work and discussion"; "guidance in the comparative merits of mathematical technology and software"; and identification of "the origins of problems students have" and means to address them (p. 1005). They recommend that there should be induction courses dedicated to mathematics lecturers specifically addressing these issues.

4. Summary and looking ahead

It is the major goal of service mathematics to make students mathematically competent (or “mature”) for their study course and later work life but both are moving targets, so it is a permanent task to follow up changes in requirements. The last 10-15 years have seen remarkable progress in understanding such requirements and in developing corresponding teaching and learning interventions even if progress often occurs in form of deeper understanding of just “isolated spots” within the overall landscape.

Part of the research was concerned with the topic of integrating mathematics education into an application study course, an issue that is specific to service mathematics. Research has considerably improved our understanding of requirements for theory development and for solving tasks particularly in engineering application subjects like engineering mechanics or fundamentals of electronics. Identification of requirements should form the basis of providing corresponding offerings in mathematics education in order to enable students to perform successfully in these subjects. But still, only a few subjects have been covered, and hence this is an important area for further work which also holds for workplace requirements where even less research is available. Attempts to include educational offerings in form of application-oriented assignments and projects of different scale (up to modelling and problem solving classes) have been made and investigated, and positive consequences for attitude and competencies have been reported. It is the intention of curricular integration to let students experience mathematics as an integral part of the study course with high relevance shaping their beliefs and behaviour correspondingly and possibly improving the ability to make use of mathematics later on.

Within the set of mathematical competencies identified by Niss, the modelling competency has received particular attention within research. It turned out to be considered as being of particular importance by engineering faculty (Faulkner et al. 2019). Despite its high value, it is still an open question where components of the modelling competency are or should be acquired (Gainsburg 2017). How the split of responsibility for developing mathematical competence between mathematics education and application subjects should be conceptualised is open and no explicit common understanding between mathematics and application study course faculty seems to be available. One might conjecture that in first-year mathematics education students should experience mathematical concepts in application models in order to let them recognise the relevance whereas a deeper and hopefully more sustainable understanding is acquired and strengthened in application subjects and later on in workplace experience.

Other competencies have only sparsely been researched and considered for corresponding interventions. There is some work on problem solving, using representations, communicating mathematics, and using aids and tools, but the role of mathematical reasoning in theory development and potential differences between mathematical and application reasoning are still to be investigated. The role of technology and its influence on the mathematical competencies required in service mathematics is an ongoing issue. As the study by Faulkner et al. (2019) showed, fluency with “basic mathematics” is required by engineering faculty but advanced analytic techniques might be obsolete. Where the borderline for a certain type of study course should reasonably be drawn is subject to further research and agreement between departments.

With respect to transition, many interventions have been made in practice and good practice reports are available e.g. for mathematics support centres. Investigations of effectiveness and efficiency of measures provide ambiguous results as reported in the study by Greefrath et al. (2017). Even within one type of intervention like bridging courses one might find quite different goals and in order to reasonably judge on success, interventions have to be evaluated against their intentions. More research in this area is needed for having sustainable results instead of introducing ever new interventions.

At several places within this survey it became clear that assessment is a major factor influencing student attitude and behaviour particularly in service mathematics settings. Offerings are taken serious by the majority of students only if they contribute in some way to the final grading. This is reflected in the concept of constructive alignment (Biggs & Tang 2011) which suggests to align goals, measures, and assessment. Whereas assessment of computational procedures is quite straightforward, the assessment of competencies and their aspects is less clear. There is some research on conceptualising the "progress" in competence acquisition which can be used for assessment but much more work is necessary.

Regarding teaching practice there is some professional and pedagogical literature (Treffert-Thomas & Jaworski 2015) but research is still very sparse. What is available suggests that coarse characterisations of teaching practice (like lectures) as "transmissionist" does not capture finer differences regarding explaining, questioning and involving students which are valued by students as "good teaching" (Mesa et al. 2015). Combining research on teaching practice and corresponding student learning would be of particular relevance.

Finally, for influencing mathematics service teaching on a larger scale and hence for putting research into practice which has not happened to a satisfactory degree (cf. Artigue 2016), the creation of networks and other organisational structures on the regional, national, and international level is indispensable. Scientific organisations like SEFI's Maths Working Group or ASEE's Mathematics Division play a role in this but also national centres like MatRIC, MEC, and KHDM (see section 2) or sigma in the UK (www.sigma-network.ac.uk) that organise networks, disseminate information on research results and good practice, and provide offerings for professional development dedicated to service mathematics, are important for reaching the "real world" of service mathematics teaching.

5. Literature

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