Does CAS at school help freshmen in engineering sciences?

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Abstract

We have known for a long time that students have many difficulties concerning basic skills in mathematics. Moreover during recent years students display new kinds of mathematical mistakes: they lack the recognition of patterns and structures within mathematical expressions. Examples are given. Furthermore, during recent years there has also been a change of paradigm in teaching mathematics at German schools, due to bad PISA results. The focus is now on modelling and solving problems in the "real world". CAS is often used. Even the final maths exams of secondary schools are written with the help of CAS calculators. As a consequence practice in applying rules (for differentiation, integration etc.) is missing.

The theses are:
The lack in basic skills is caused by using calculators too early.
The lack in applying rules is caused by using CAS too early and too much.

Symptoms

We have known for a long time that students have many difficulties concerning basic skills in mathematics. Moreover during recent years students display new kinds of mathematical mistakes: they lack the recognition of patterns and structures within mathematical expressions. Here are some examples: well known are the problems students have in extending an expression of the form x^2+3x to a complete a square. They have difficulties recognising the pattern and comparing with $a^2+2ab+b^2$ to find $(3/2)^2$ as the missing term. We have also known for a long time that the recognition of structures e.g. in a double fraction, is very weak.

In addition during recent years further problems concerning structure have arisen: for example, log, sin, cos are not identified as functions, they are treated as "normal" symbols. Calling the function $\sin(x)$ is understood as a multiplication like $\sin * x$. This becomes obvious in the following mistakes:

- Applying a kind of a commutative law for simplifying the expression: $\sin(x)\cos(x) + \cos(x)\sin(x) = 2\, x \sin \cos$,
- applying the product rule for differentiation: $d/dx\,(\sin(x)) = \cos(x) + \sin$.
- These newer kinds of mistakes give a different look to an old mistake: $\sin(x+y) = \sin(x) + \sin(y)$ can be interpreted as consequence of the distributive law.

Often in bridging courses at university students are not able to identify rules that are the basis when they transform terms. For examples, even when all rules for dealing with powers are projected on a screen, many students are not able to identify the rule behind $(-x)^2=x^2$. If they are forced to do this, a typical answer is: "I do not know, I am following always my gut feeling." But, they do not understand this observation. The students know in principle how to deal with mathematical terms. For example they know how to simplify $uv+vu$, but if the situations gets a little bit more difficult, then they get completely confused.

Reasons

If we look for the reasons behind these mistakes, we have to take into account that during recent years there has been also a change of paradigm in teaching mathematics at German schools, due to bad PISA results. The focus is now on modelling and solving problems in the “real world”. CAS is often used and CAS is a necessary tool for it. Even the final maths exams at many secondary schools in Berlin are written with the help of CAS calculators. It is the aim that all secondary schools in Berlin will do so.

As a consequence the computer does the differentiation and integration and not the student. Thus an opportunity to practise applying rules and therefore an opportunity to practise recognizing structures has gone. The reduction of practice at school has also to do with the fact that the focus of the education is on subjects that the majority of students at school will need in their future jobs. Arnold a Campo (2007), the head of the MNU (Deutscher Verein zur Förderung des mathematischen und naturwissen-schaftlichen Unterrichts e.V., Society to promote mathematics and science education), states that only a small percentage of the students at school will become an engineer or a scientist.

For a long time it has been common that normal calculators are used at school. The usage of calculators starts at least after the first six years of school, just when fractions are introduced. The consequence is a lack of basic skills like dealing with fractions and later with symbolic fractions. Today the increased use of CAS will play a similar role concerning recognition of structures.

The change of paradigm of teaching mathematics at German schools had started earlier, at the end to the eighties, in the Netherlands. Krieg, Verhulst and Walcher reported about a protest of students in the Netherlands against the low level of mathematics teaching at school. They expressed their alarm in a public letter to the minister of education, Maria van der Hoeven, signed by 10 000 students. This letter has become popular under the motto “Lieve Maria”.

We also must take into account that student’ life, and our daily life in general, has changed. There is a flood of information. Looking for solutions is now a quick and impatient online search. New devices like digital cameras, mobile phones etc. must be self-explanatory without long instructions. The young generation can be regarded as natives of our digital world, while the older generation, born in the non-digital world, are immigrants. The natives of the digital world are experts in the trial-and-error-method and in looking for answers just by one click. But they are not trained for a time-consuming, systematic and deductive acquisition of knowledge. They are not well trained to follow formal rules of either a natural language or a mathematical language. The digital world promotes more intuitive than rational competencies.

Role of practice

Elsbeth Stern is an expert in cognitive psychology at the ETH Zürich, Switzerland. Stern (2006) emphasises the role of practice. "A person, who is not experienced in reading, has to transform every letter into a phoneme and has arduously to construct a word out of it. RAM capacity is occupied that is lost for understanding the content." [Stern]. Our engineering freshmen have comparable difficulties. They are not experienced in reading formulas, they spell the mathematical expressions. Thus RAM capacity is bound that goes lost to capture the meaning and the structure of the expression. Like reading, doing the basics in mathematics has to be automated. Automating releases RAM capacity, this is needed for the creative process of understanding and problem solving and saving new information.

"Discussions with staff led to the identification of some specific mathematics content viewed as exceptionally difficult for learning. These include spatial visualisation, algebraic manipulations, abstract notion, complex numbers and mechanics. It is noted that elements of such content depict more mathematical skills and application than knowledge." [Mansons, Alpay, 2010]

What to do?

Primarily the students have to be forced to train their mathematical skills. As a consequence the usage of computers in the exercise lessons has to be reduced. Also a part of the written assessments should be done without the help of calculators.

Twenty years ago the Beuth University of Applied Sciences established a bridging course. It takes place in a compact form just in the last eight days before the freshmen will start their lectures. The participants form groups of about no more than forty. So a kind of individual help is possible. The problem is that eight days of exercising are not enough to substitute for fewer practices that were not developed during twelve years in school. Two further remedial actions at Beuth University have been established: an online bridging course and "L+". The online bridging course is available for all students at all times, it is available before the beginning of, as well as during, their studies. "L +" means "Learning plus", it is a one-to-one help system. Members of the teaching staff (retired and temporarily teachers) guide students to help themselves. L+ is offered not only for mathematics but also for other subjects like mechanics.

There are also problems other than the lack of basic skills within the mathematical education of engineers: the motivation of students and the ability of linking and recognising mathematics in the subjects of the engineers. To overcome these problems a careful setup of the curriculum and permanent time-consuming discussions among the concerned teaching stuff is needed. Also integrating project works taken from the engineering subjects into the mathematics education helps. Alpers (2006) and Diercksen (2008) gave convincing examples for mechanics and for electrical and electronic engineering, respectively.

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