Image processing to Motivate Linear Algebra Students

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Abstract

The increasing development of computer technologies has given rise to educational alternatives, facilitating the creation of new forms of learning in an attractive and motivating way. Teachers develop efforts to create learning techniques using methods that can facilitate the learning in different areas of knowledge. The purpose of this experiment is to describe a case study that the author made with first year students of Electromechanical and Mechanics Engineering in a Linear Algebra course at the Coimbra Institute of Engineering. The study involved the application of Digital Image Processing to teaching and learning the basic concepts of Linear Algebra. The experience was very enriching for the author as a teacher, as well as for the students, because it let them awaken the motivation to learn, the interest and taste for Linear Algebra.

Introduction

Arousing curiosity and motivation in students is not always an easy task. To motivate students is necessary as the content or activities enrich experiences that encourage student interest and curiosity. Because not all students learn in the same way, the teacher makes content more engaging and motivates the students to perform the various activities with interest. Thus, the use of different methodologies can contribute to motivating students to find more meaning in what is expected of them. Through this motivation, students find reasons to learn and improve all their skills. This explains the concern of teachers, specifically teachers of mathematics, in seeking to motivate their students and get positive results, reducing the failure of mathematics.

Investigations on methodologies for teaching strategies focus on identifying different types of learning and motivation of students associated with this learning [Bulut (2011), Habash (2010), Meece (2006)]. The use of technology and examples of application of Linear Algebra help enrich the traditional methodology in order to facilitate and encourage student learning (Berriochoa (2009), Caridade (2011)). The combination of learning with more explorative activities enables students to acquire concepts, and terminology while developing self-confidence in using mathematics (Silverman(2010)). Some experiments have been made: the use of the Web to improve student learning (Waldock(2002)), the development of multimedia applications laboratory activities to develop mathematical concepts of trigonometry and pre-calculus (Rosen(2005)), the ATLAS project to encourage and facilitate the use of software in teaching Linear Algebra (Roberts(1996)) and the programme of research on teaching and learning of Linear Algebra in the first year of science courses in French universities (Dorier(2000)).

The teaching of Linear Algebra and Digital Image Processing

Linear Algebra is taught to a large and diverse number of students. Who are the students of the Linear Algebra course? How to motivate these students? What is the best way for students to acquire knowledge and skills in this area?
MATLAB (Mathworks (2002)), is a computer algebra system, designed for professional use in solving problems that require mathematical methods. It is one of the packages for the most natural application of Linear Algebra, since it is specially developed for matrix operations. Digital Image Processing is a highly relevant field, extremely rich in mathematical ideas that allow learning of Linear Algebra in a way completely different from the standard one.

In this context, I intend to present my experience to interest and motivate students to develop skills using a methodology that allows interdisciplinary between Linear Algebra (LA) and Digital Image Processing (DIP). This experience was developed in October 2011, at the first LA lectures with 40 students of Electromechanical Engineering and 73 students of Mechanical Engineering at Coimbra Institute of Engineering. The theoretical concepts of LA were presented by the DIP applied to images. This study is based on the following objectives: enable the acquisition of LA knowledge; develop reasoning and critical thinking of the students; encourage self-learning; increase student motivation in LA.

**Development activities**

One of the first concepts that student get exposed to in LA course is matrix operations. Let $A$ and $B$ be two matrices (images) with the same dimension $m \times n$. The matrix obtained by adding (or subtraction) the previous matrices $A$ and $B$, called $A+B$ (or $A-B$) is shown in Figure 1 by the addition/subtraction some parts of the smiley face.

\[
A \pm B = \begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mn} \\
\end{bmatrix} \pm \begin{bmatrix}
  b_{11} & b_{12} & \cdots & b_{1n} \\
  b_{21} & b_{22} & \cdots & b_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  b_{m1} & b_{m2} & \cdots & b_{mn} \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  a_{11} \pm b_{11} & a_{12} \pm b_{12} & \cdots & a_{1n} \pm b_{1n} \\
  a_{21} \pm b_{21} & a_{22} \pm b_{22} & \cdots & a_{2n} \pm b_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} \pm b_{m1} & a_{m2} \pm b_{m2} & \cdots & a_{mn} \pm b_{mn} \\
\end{bmatrix}
\]

Figure 1. Addition and subtraction of two matrices.

The scalar multiplication of $k \in \mathbb{R}$ by a matrix $A$, represented in Figure 2 can be teaching by DIP, using the increase or decrease light image. In the case represented in Figure 2, the image of the wolf in the centre ($2A$) was darker and the image on the right ($0.5A$) was lighter than the original one, on left.
Figure 2. Scalar multiplication of one matrix.

The transpose of a matrix $A$ is another matrix $A'$ which is formed by turning all the rows of matrix $A$ into columns and vice versa. An example of this operation is presented in Figure 3. The image of a flower is transpose. The students can see the difference between a transpose image and a rotate image by 90 degrees in positive or negative angle.

Figure 3. Transpose a matrix.

The multiplication of two matrices $A$ and $B$, are represented in Figure 4. In the top right, the image represents the multiplication of a starfish by matrix $I'$ on the left, and the bottom image represent the same multiplication but on the right. In the first case the starfish is a horizontal reflection of the original image and in the second a vertical reflection.

Figure 4. Multiplication of two matrices.
The properties of the matrix operations can also be operated by the application of DIP images. For example in Figure 4 it is possible to illustrate that the multiplication of matrices are not commutative \((A \times B \neq B \times A)\). Another example of the properties is the transpose of the sum is the sum of the transposed matrices \(((A + B)^T = A^T + B^T)\) that are represented in Figure 5. The transpose of the image \(A + B\) is equal to the sum of the image \(A^T\) with \(B^T\).

![Figure 5. The transpose of a sum of two images.](image)

Another context that the student can explore in LA classes are geometric transformations applied to images. Let \(A\) be a matrix of dimension \(3 \times 3\) and \(u\) a column matrix of dimension \(3 \times 1\). A transformation matrix of the function \(f: \mathbb{R}^3 \rightarrow \mathbb{R}^3\) is defined by \(f(u) = Au\), where \(u\) represents a pixel and \(A\) represents a matrix of scaling, rotation or translation. In Figure 6 is represented some geometric transforms applied to different images.

![Figure 6. Geometric transformations applied to images.](image)

The left image represent a scaling of the Eiffel Tower, the centre image a rotation of the Pizza Tower and the right image a translation of Coimbra Tower. In these examples the matrix \(A\) is applied at all the pixels on the image.

**Results and Discussion**

All forms of manipulating images (to improve contrast, the increase or decrease light, etc.) may be performed by applying operations to mathematical matrices associated with
each image. A natural link between LA and DIP, supported by computational tools and contemporary technologies can be explored in the LA course.

My teaching experience has shown that the traditional methodologies are no longer responding to the new educational paradigm. The students eager to learn, and especially to know how to do, require dynamic new and practical activities of learning where they are the centre of the teaching-learning process. The teacher was the transmitter of knowledge and has become the guide of learning, whose objective focuses on the student and in the construction of their learning. Thus, education should be motivating and exciting for student learning, and have meaning. In this sense, use of MATLAB software and applications of DIP in LA courses allow a considerable improvement in student learning.

The experience described is performed for the purpose of determining whether the use of this new methodology in the classroom increases student learning. It always has, as its main objective, to captivate students' interest throughout the lesson, providing an environment of teaching and learning enjoyable and motivating. Students cooperated and answered the questionnaire that I proposed, showing great interest and curiosity, mainly by reference to DIP. The programme content learned in most of the first students was facilitated by using the relationship between matrices and images. It was evident that the experience was very enriching for the students; it led them to think more consciously about matrices and their properties; and was enriching for me as a teacher.

At the end of the lesson, students completed a survey about their interest in the new methodology. Some of the answers are presented below:

1. The activity performed with the Image Processing was interesting?

“Yes, it was a funny and interesting way to understanding the operations between matrices”; “I found it very enlightening and appealing”; “It was interesting, motivating and easier to understand”.

2. What is your assessment of the activity performed with the Image Processing?

“This type of practical activity has the advantage to make the students more interested in Linear Algebra classes”; “An activity that arouses a great interest in the subject and in matter itself”; “Positive. Interesting until the end”; “I think it was not possible to improve”.

Conclusions

This experience was very positive, in the sense that the introduction of this methodology helped to awaken the motivation to learn, interest and taste in LA. We all benefited by developing this kind of mathematical activity. Students were able to learn mathematics in an engaging way, as a tool that facilitated their learning. The teacher can see their students motivated and interesting with the entire context presented in lessons.
Although the experience has been limited to some content, I believe the importance of this type of methodology justifies its presentation. By the way, it is evident from the reference of students that learning is more exciting when using a different methodology that can motivate the interest of the students. This can be seen by the feedback from students. Most of the students enjoy the experience, join it and they were very active. In the next school year I can extend the project to other contents of LA, such as the calculation of eigenvalues and eigenvectors.

References


