Using a computer-animated graphical approach to teaching differential equations

Imre Kocsis

Faculty of Engineering, University of Debrecen, Debrecen, Hungary

Abstract

We give a short survey on a method applied in the training of engineers to increase the effectiveness of the teaching process in the field of differential equations. The basic ideas of the method are to show the connection between theory and practice via carefully selected examples and to give visual information in connection with the concepts studied to help motivate the students. Sequentially, we propose a technical problem with a differential equation in the background, we determine a parameter which has an essential role in the solution to the problem and finally, using animation, we visualize the dependence of the solution function of the differential equation on this parameter. An approximate solution to the problem is obtained from following the steps of the animation.

Introduction

The effectiveness of a course in mathematics can be characterized by the students' ability to discover the different occurrences of concepts and ideas in the investigation of technical problems and to apply mathematical methods to solve them ((Kocsis, Sauerbier & Tiba (2006)).

It is difficult for the majority of students to find the connection between the mathematical language and the technical way of thinking. One of the disadvantages of classical blackboard-based presentation technique that the possibilities for visualization are limited, while it is widely accepted that visualization is a powerful tool in the learning process of engineering subjects (Park & Gittelman (1992)). Up-to-date IT helps us to find new methods and suitable approaches to bridge this difficulty.

Well-planned representations assisted by special software may have a strong impact on the lectures, showing the concrete meaning of the concepts discussed in a course. Combining the abstract concepts with the practical technical problems can also increase the effectiveness. Our example shows a possible way.

An example

The background of the example

In the subject of second order differential equations, we can investigate as an example the second order differential equation $y''(x) = \sqrt{1 + y'(x)^2}$. Students can learn the solution method and can determine the solution functions, but limiting ourselves to calculations in the mathematical model is not motivating enough for them (Kocsis (2007)). At this point we can present the well known problem of a flexible heavy cord fixed at its ends. If $x \to y(x)$ is the function describing the shape of the heavy cord as shown in Fig.1, it is known that with the assumption y’(0)=0 the function y satisfies the second order (nonlinear) differential equation

$$y''(x) = k \cdot \sqrt{1 + y'(x)^2} \ ,$$

where k>0 is a parameter depending on geometrical and physical characteristic data. By this interpretation the students can visualise the investigation of the equation from technical a point of view.

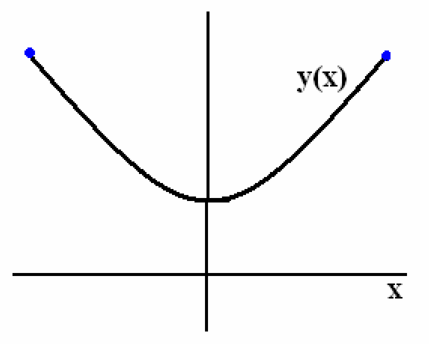

Fig. 1 Heavy cord fixed at its ends

Presentation of a more concrete problem in connection with the investigated differential equation

Our aim is to show the connection between theory and practice. To do this we continue reducing the level of abstraction. In our experience, the following example is concrete enough to motivate the learners. Consider the ends of a flexible heavy cord at two points at the same height with 10 [m] distance between them, as shown in Fig. 2. Suppose that the maximum admitted dip is 3 [m]. Determine the maximum length of the cord.

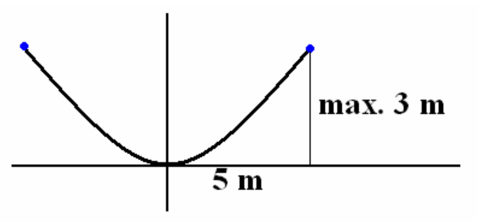

Fig. 2 Sketch of the problem

A possible way to determine the answer is first solving the differential equation (with or without computer) and investigating the cosine hyperbolic function provided as a solution. In this case a numeric method is needed to solve a hyperbolic equation, and finally we get an approximate answer to our question.

Here we follow another way. We use animation (created by Maple) to get an approximate value for the maximum length of the cord.

Solving the problem with animation

We manipulate with two Maple commands; **`DEplot`** to provide the graph of the solution function with given parameter value for k, while **`animate`** generates a series of pictures according to the changing value of the parameter k. To study our problem we are change the value of k and check the value of the solution function at x=5, y(5).

Increasing the value of parameter k in the differential equation increases the value of y(5). We fix the increment size (ε>0) and run the animation with values k = ε, 2ε, 3ε, .etc. Controlling the display of graphs we can determine an approximate maximum value of k satisfying y(5)≤3. It is clear that the smaller ε is, the more accurate the approximation is.

Simplifying the calculation we assume that y(0)=0 [m], that is, we consider the following initial value problem:

$$y''(x) = k\sqrt{1+y'(x)^2}\,,\qquad y(0)=0,\qquad y'(0)=0$$

on the domain $x\in[-5,5]$.

To construct the animation we apply the following MAPLE commands

```
> restart:with(plots):with(DEtools):
> DE:=diff(y(x),x$2)=k*sqrt(1+(diff(y(x),x)^2));
> animate(DEplot,[DE,y(x),x=-5..5,[[y(0)=0,D(y)(0)=0]],
linecolor=black,stepsize=0.1],k=0..0.3,frames=31,background=
plot(3,t=-5..5,colour=blue,thickness=3));
```

The value of k is changed gradually in increments of 0.01 and Figures 3-6 show some of the results. We can see four steps of the animation: k_1=1, k_2=1.5, k_3=2, and k_4=2.2 and estimate that k_{max}=2.2, approximately.

From the maximum value of k (k_{max}) we can calculate the maximum length of the cord (L_{max}) using the following relation [1]:

$$s^2+\left(\frac{1}{k}\right)^2=\left(y(x)+\frac{1}{k}\right)^2$$

where y is the solution function (the shape of the cord) for the parameter value k, and s is the length of curve AB. (Fig.7)

Thus we have

$$L_{max}=2s_{max}=2\cdot\sqrt{\left(y(x)+\frac{1}{k}\right)^2-\left(\frac{1}{k}\right)^2}=2\cdot\sqrt{\left(3+\frac{1}{0.22}\right)^2-\left(\frac{1}{0.22}\right)^2}=12.04$$

So the maximum length of the cord is L_{max}=12.04 [m].

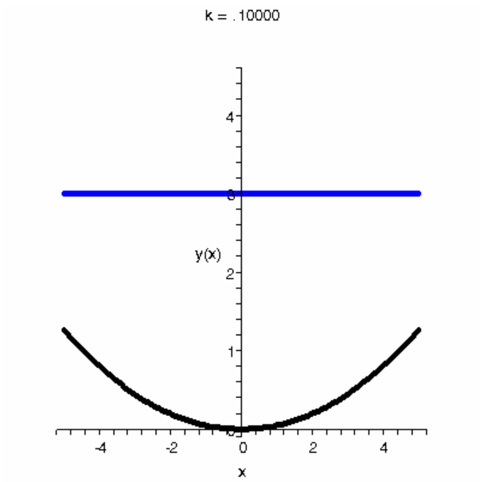

Fig.3 The solution (k=1)

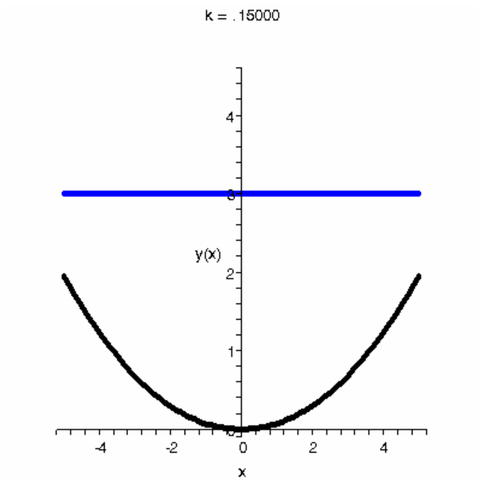

Fig.4 The solution (k=1.5)

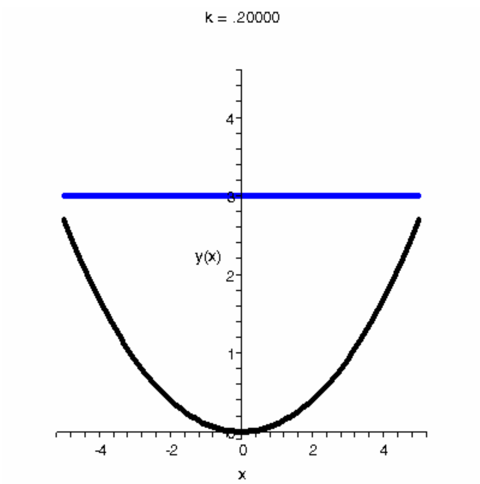

Fig.5 The solution (k=2)

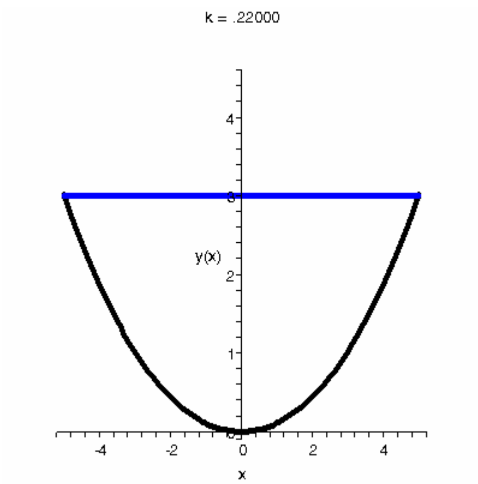

Fig.6 The solution (k=2.2)

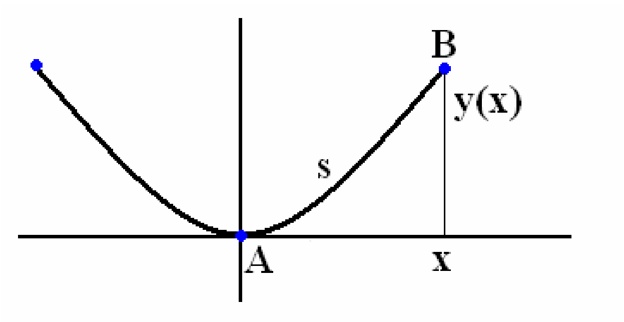

Fig.7 Calculation of the length of the cord

Conclusion

In our opinion the special facilities provided by computer needs new approaches in teaching mathematical subjects. There are topics where animation can greatly increase the effectiveness of the teaching process. It is especially true in the theory of differential equations.

References

Fazekas, F. (ed.) (1973) *Műszaki matematikai gyakorlatok B.VII. Közönséges differencálegyenletek (második rész).* Tankönyvkiadó, Budapest, pp. 156-158.

Kocsis, I. (2007) Application of MAPLE ODE Analyzer in the investigation of differential equations in the higher engineering education. *Pollack Periodica*, Vol. 2.: 177-183.

Kocsis, I., Sauerbier, G. & Tiba, Z. (2006) Die Nutzung des Computersystems MAPLE in der Ingenieurausbildung der Universität Debrecen. *UICEE Global Journal for Engineering Education*, Vol. 10.: 287-292.

Park, O. & Gittelman, S. S. (1992) Selective use of animation and feedback in computer-based instruction. *Educational Technology, Research and Development*, 40(4): 27 — 38.