

Testing and Teaching Mathematics with Interactive Online Technology

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Abstract

First year engineering students in Finnish polytechnics have great motivation problems and very variable backgrounds. This leads to a high drop-out percentage and low study achievements. First year problems tend to concentrate on theoretical studies, especially mathematics. The Information Technology programme in Helsinki Polytechnic has decided to give special emphasis to the start of the studies by placing the students in small groups appropriate to their school background and their mathematical abilities.

The engineering faculty has started using an interactive online mathematics testing and teaching system for the first year students. The system is based on algorithmically generated numerical and symbolic problems and it gives immediate grading and detailed feedback for students. The system is used by all students commencing information technology and industrial management. The system is also used in remedial mathematics courses in other engineering programs.

The diagnostic tests of the system are used to assess the level of basic mathematical skills of the new students, both for students themselves and instructors, and also to place students in appropriate study groups. The results show that the diagnostic test correlates well with the achievements in the mathematics class in the first study period.

In industrial management we have used the system also for continuous online self-tests and homework assignments. Students use the system regularly and they are using its feedback to monitor their mathematical progress, with minimal intervention by the teacher. The results in the assignment system and the tests correlate even better with the achievements in a regular class test.

Why Automatic Testing and Assignment Systems?

First Year Study and Motivation Problems

The Finnish educational system provides higher education for about two thirds of all young people. This means that universities and other higher education institutes compete heavily with each other for good students. The competition has been especially detrimental to engineering departments in polytechnics (universities of applied sciences).

Young people applying for engineering studies in polytechnics have very varying backgrounds and often great motivation problems. Roughly one third of the accepted students have a senior high school background with extensive mathematics and some physics. They have, however, mediocre grades in mathematics on the average, the best students going to universities. The other thirds of the accepted students have studied only general mathematics and no physics in the senior high school or they come from vocational schools. These groups take usually somewhat slower and broader mathematics courses during the first semesters, but even then they have lower study achievements.

First year study problems concentrate on theoretical studies, especially mathematics, which is the essential basis for serious engineering studies. Young people who have had little success in mathematics and other theoretical studies in school would rather study something else, especially something more practical or application oriented. This, together with other reasons, leads to low motivation and failures and high drop-out percentages, which have lately been from 30% to 50%, highest in information technology, in the Helsinki Polytechnic engineering study programs.

The information technology program has tried to deal with the first year study problems taking several different measures. In addition to special tutoring, students have been placed in small study groups (20-25 students) according to their mathematical abilities. They use a special supporting pillar system, and they have ample opportunities to do online mathematics problems and self-tests getting immediate feedback. They are also tested in the beginning and after the first study period.

Diagnostic and Placement Tests

Information technology and Industrial management students (240 students yearly) take a basic mathematics online test just before the actual studies start. The test consists of 35 problems in algebra, equations, trigonometry and functions. The problems are parametrically and randomly generated in real time from carefully selected problem prototypes, which means that there are always different problems in each test or assignment set. The testing system is based on the MapleTA computer aided assessment (CAA) system and it is provided by WebALT Inc.

The screenshot shows a web browser window with the URL `http://www.webalt.com:8081 - Stadian tentti - harjoituksia, Algebra1 - Mozilla Firefox`. The page title is "WebALT Graded Session - Credit awarded" and it features the MapleSoft logo. Navigation buttons include "Back", "Next", "Jump To:", "Grade", "Help", and "Quit & Save". The main content area displays "Stadian tentti - harjoituksia, Algebra1" and "Question 5 of 5" with "Remaining time: Unlimited".

Question 5: (1 point)
Sievenna murtolauseke

$$\frac{3a}{36a^2 + 12a}$$

Input field: `1/(12*a + 4)`

Below the input field, there are links for "Plot", "Help", "Change Math Entry Mode", "Preview", "Hint 1", and "Hint 2".

A preview window is overlaid on the right, showing the fraction $\frac{1}{12a + 4}$ and a "Close" button. The status bar at the bottom of the browser window says "Done".

Figure 1. A test item ('simplify a fraction') and a student's answer. The Preview window is also shown.

Tests are used to diagnose the level of basic mathematics for each student. The test result and school background are used to place the student in an appropriate study group. Feedback gives teachers information on what can be expected and which

remedial actions must be done. Students get detailed feedback that includes automatically generated detailed solutions.

The test is repeated after the first study period (8 weeks). This gives information on how the basic mathematical skills of individual students and study groups have developed. Automatic tests can be used to measure progress and the effect of various pedagogical and other interventions. However, it is essential to validate the automatic tests by comparing results with other tests and grading methods.

Supporting Student Work

The same MapleTA system is used for automatic online homework assignments and continuous self-testing. A homework assignment set typically consists of about 5 problems that are related to the topic students are currently studying. There are typically 20-30 assignment sets for an eight-week mathematics module. The assignments can be compulsory and integrated into course pedagogy, or they can be voluntary and independent, depending on the teacher.

The screenshot shows a web browser window with the URL <http://www.webait.com:8081>. The page title is "Maple T.A. 2.51 - In-Session Feedback - Stadian tentti - ...". The MapleTA logo is visible in the top left, and a "Close" button is in the top right.

Question 3: Score 0/1

Laske murtolausekkeiden $\frac{6a}{5y-10}$ ja $\frac{3a}{y-2}$ osamaara. X
 Ilmoita vastaus murtolukuna. INCORRECT

Your Answer: $(2y-4)/(5y-10)$
Correct Answer: $2/5$

Murtoluku jaetaan murtoluvulla siten, että jaettava kerrotaan jakajan kaanteisluvulla.

$$\frac{6a}{5y-10} : \frac{3a}{y-2} = \frac{6a}{5y-10} \cdot \frac{y-2}{3a}$$

Jaetaan tekijöihin, jotta saadaan sievennettyä.

Comment:

$$= \frac{6a}{5y-10} \cdot \frac{y-2}{3a}$$

$$= \frac{6a}{5(y-2)} \cdot \frac{y-2}{3a} \text{ (supistetaan } (y-2) \text{ :lla ja } 3a \text{ :lla)}$$

$$= \frac{2}{5}$$

Figure 2. Feedback from a problem (division of two fractions) with intermediate stages and comments. The student's answer is not accepted because it has not been simplified enough.

Feedback from each problem gives students valuable and critical support for self-study. The automatic feedback is immediate and detailed and it is available online whenever the student has time to do the assignments. Students can monitor their progress, with minimal intervention by the teacher.

Remedial Courses

Due to the first-year difficulties, there are large numbers of students who have failed their first-year mathematics courses, despite several attempts. Since many students are about to graduate and some of them are already in work, they are not especially motivated to take the basic course in school again. For this reason there is a possibility of wastage even at this point of studies.

The electrical, mechanical and automotive engineering programs have decided to give special first year mathematics remedial courses for students that are otherwise in advanced stages in their studies. The courses are based on self-study, a few tutorials (partly online), and compulsory online MapleTA assignments. The exam, however, is a conventional supervised written exam. Students have found this arrangement very motivating.

Results Found

Diagnostic Tests and School Background

Traditionally students have been placed in parallel study groups according to their school background, namely senior high school with extensive mathematics, senior high school with general (short) mathematics, or vocational school.

The diagnostic tests show that there is substantial overlap in the mathematical abilities of the three groups and a high variance in each group. Therefore we have decided to use both the school background and the test result as a basis to place the students in parallel study groups.

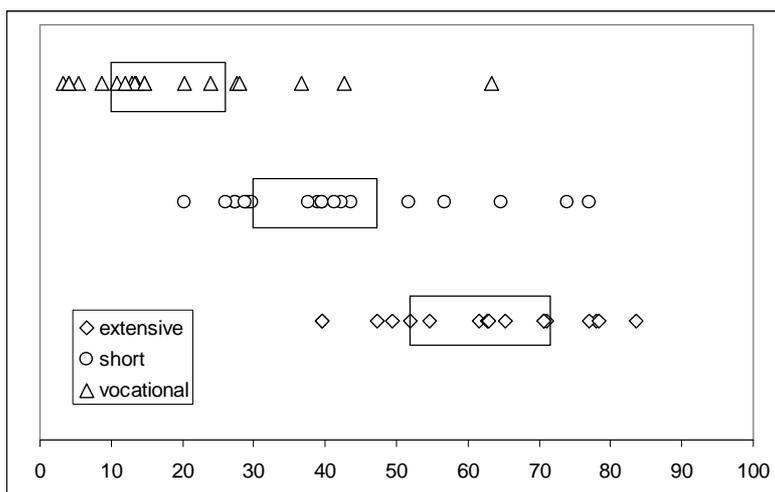


Figure 3. Diagnostic test results and school background.

Predicting Achievements

Results of the regular class test after the first study period have been compared to the results of the diagnostic tests and other factors that might explain the results of the class tests (which are the basis for the course grade). We have found that the diagnostic tests have consistently high correlations with the class tests. Similar results have been found elsewhere, e.g. Heck & van Gastel (2006a, b). For those groups that have used the MapleTA assignments during the first period, the assignments activity has even higher correlation with the class test. Similar results have been found also in the Helsinki University of Technology (Rasila, Harjula & Zenger (2007)) and in Finnish primary schools (Lehtinen (2008)).

	Diagnostic Test 1	Diagnostic Test 2	Online assignments	Regular assignments	Class attendance
Class test	0.63	0.69	0.73	0.59	0.39

Table 1. Correlations of diagnostic tests and other factors with regular class tests. N = 70, two separate study groups.

Effect of Assignments

The impact of the MapleTA assignments integrated into the first period mathematics course is clearly seen when we compare the average group test scores of a group that uses assignments with groups that do not use the assignments. For those who used the assignments the average score improved by 35 points, but for those who did not use the assignments the score improved only 18 points on the average (see Figure 4).

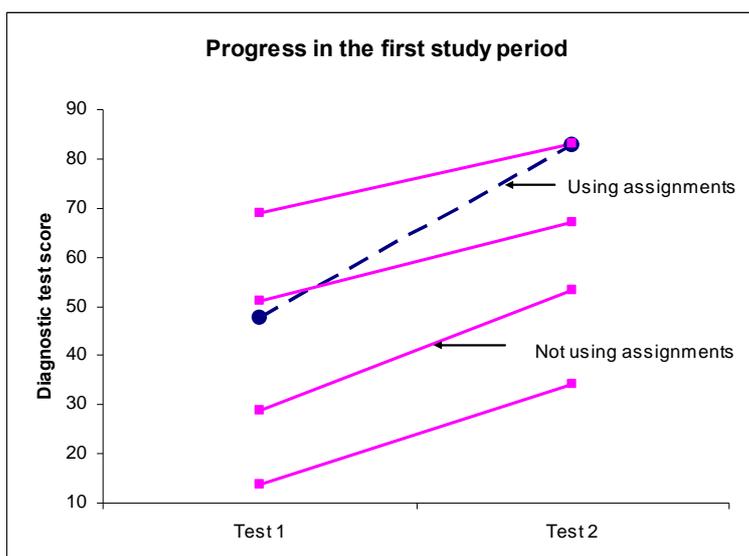


Figure 4. The effect of MapleTA assignments to the development of diagnostic test scores.

Practice and Feedback

Although the MapleTA system is quite straightforward to use and easy to learn, there is however a threshold for its use. Generally, students do not take any assignments voluntarily. The assignments should be integrated into the course. With some initial encouragement and push, students will use the system independently and actively and they start to see its benefits. During one eight-week study period the Helsinki Polytechnic Industrial management students did, on average, 3.7 assignment sets per week. To get good scores many students did the same set over and over. As the assignment problems are always different, this promotes the learning of general principles, not just memorizing the right answers. The feedback is generally positive, while the usability and consistency of the system must be improved.

Discussion

To conclude, we maintain that the main benefits of computer aided test and assignment systems are:

- To place students in proper study groups, together with other criteria.
- To minimize the number of drop-outs.
- To minimize unnecessary review courses.
- To construct an effective and economic study track.
- To support and speed up students' self-study and practice.
- To measure the effect of various pedagogical and other interventions.

Automatic testing might augment or possibly replace traditional testing in mathematics. Both synchronic and diachronic comparisons become possible because the same standardized tests can be used over and over.

References

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