Using the SONG approach to teaching mathematics

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Abstract

In 2003, as part of the UK MathsTeam project, we presented a case study illustrating our way of encouraging engineering students to learn mathematics. This is based on SONG, a mix of Symbolic, Oral, Numerical and Graphical approaches. Students are thus exposed to multiple representations of the mathematical concepts they need to understand, exploiting in a variety of ways the technology-rich context in which they find themselves. Since that project the world, and our university, have inevitably moved on. We must now teach CEng, IEng and a smattering of Foundation degree students together in the same group, and this increased diversity exacerbates some of the difficulties, such as the range of mathematical backgrounds, and the varying levels of confidence, skill and motivation, which we and our colleagues in the UK mathematical community have been facing.
Given the increasingly diverse backgrounds, where in many cases students have struggled with mathematics; can we give the chance of a fresh approach rather than just revisiting their previous difficulties?
In a technology-rich environment, with multiple representations and multiple independent ways of getting answers, arguably no student should ever get any sums wrong; but what is the best way to use this technological power to motivate students whatever their approach to learning?

Introduction - can things only get better?

When we were one team amongst many in the UK MathsTeam project (LTSN (2003)), we presented our ideas and practice in teaching mathematics to engineering students, one aspect of which was to exploit mathematical technology to allow multiple representations of mathematical ideas and concepts. Our acronym was SONG, a reminder that we use a mix of Symbolic, Oral, Numerical and Graphical approaches, rather than over-emphasising say an algebraic approach.

We might ask in what ways things have moved on since then. It is useful to consider the wider context within which we live and work, and from which we draw our students. The recent Leitch Report (HM Treasury (2006)) states that despite having made good progress over the last decade, aspects of the UK skills base remain weaker than those in other developed economies, for example:

- 5 million adults in the UK lack functional literacy;
- 17 million adults in the UK have difficulty with numbers.

In terms of factors which might affect these figures in future, more than one in six young people leave school unable to read, write or add up properly. The report notes that such low skills levels, if not addressed, will inevitably not only damage business, but will also result in increasing inequality. The figures describe part of the context from which we draw our engineering students, and betray something of the mathematical tone in the country. We do of course have some control over those we admit to our courses, but economic pressures on universities, and indeed the need to provide a qualified engineering workforce, are significant factors. We select those who are more likely to be competent, but we do teach an increasingly broad range of levels and abilities. Thus those teaching elite courses may be able to rely upon a certain level of competence, although even in that case the difficulties with identifying exactly what can be expected upon entry have been well explored over the last few years. The situation with lower level courses, such as preparatory years or Foundation degree (FdSc) entry years is even less predictable, and we do find ourselves addressing issues of what we would like to treat as the basic numerical awareness we would hope all citizens might achieve.

In our own context, we must now teach CEng, IEng and a smattering of Foundation degree students together in the same group, and this increased diversity exacerbates the difficulties, such as the range of mathematical backgrounds, and the varying levels of confidence, skill and motivation.

While we do operate the standard “sticking plaster” measures, such as extra classes for selected students, and a drop-in Maths Help, we find we need to constantly examine what wider educational factors we take into account when designing learning activities.

Skills and Learning as well as Mathematics

What skills? What learners?

We are not teaching mathematics for mathematics sake, even though in our enthusiastic hearts we believe that is an intrinsically valuable thing to do! We are teaching mathematical ideas so that our students can become competent engineers. Mathematics must seem like an integral part of an engineering student’s course, not a parallel activity. It therefore behoves us to know about the generalities of engineering education. This means not only possessing an awareness of the range of applicability of the mathematics we teach, but also those more general skills that the engineering profession itself says that it values and wishes to develop in its members. We have written about this previously in a context of both engineering and mathematics (e.g. Challis et al. (2002)).

A glance at the HEA Engineering Subject Network (HEA Engineering Subject Network (2008)) reveals a familiar story. Still the most commonly valued generic key skills are communication, literacy, numeracy, team work, problem solving, information technology (IT) and self-management. Integrating the development of these skills into mathematical activity can help both in providing freshness of approach to mathematics, and also in integrating mathematics into the rest of a student’s engineering course.

Moving on to learners, we once gave a talk with unofficial subtitle “Mathematicians are people too”. The same probably applies to engineers. As mathematicians we are very good at designing coherent curricula, with clear pre-requisite structure and mathematical inter-relationships and appropriate content. But do we design our range of learning activities to take account of the range of learners that we encounter?

What follows in the next few paragraphs contributes to the debate about the relationship between mathematicians and mathematical educators, the currency of which is indicated by activity within the HEA Mathematics, Statistics and Operational Research Subject Network on precisely this topic. With some noble exceptions, this relationship seems at best to achieve mutual toleration, and yet if each community could free itself from its jargon, we have much to learn from each other.

One area in which this is the case is in achieving awareness that different students have different ways of learning, and that it is useful to recognise that in designing learning activities. There are various sources which can help in developing that awareness (e.g. Support4Learning (2008) or Felder & Solomon (2008)). Categorisations are many, so one must not be too dogmatic or definitive about this. Also it is worth noting that no one person always behaves as one type of learner, and perhaps the best learners adopt different styles as the occasion demands. But nevertheless the ideas can be thought-provoking.

For instance, Felder and Solomon (2008) describe active learners as those who “tend to retain and understand information best by *doing* something active”. They say one of their phrases is “Let’s try it out and see how it works". Reflective learners, on the other hand, “prefer to think about it quietly first”. Significantly for much of our practice, they propose that “sitting through lectures without getting to do anything physical but take notes is hard for both learning types!

They draw a different contrast between visual learners, who (obviously) like pictures; and verbal learners, who prefer descriptions and words, but they propose that “all learn more when information is presented both visually and verbally”. Is there a parallel here with the historical development of our own subject, as developments have swung between algebraic and geometric approaches?

As a third and final example, Felder and Solomon distinguish between sequential learners who progress in small linear steps (the “inchworm”), and global learners, who tend to progress in large jumps, and suddenly “get it” (the “grasshopper”).

We should aim to switch on to mathematics, students exhibiting all these traits. We can use any means which comes to hand. One possibility lies in integrating basic skills development into our mathematical developments, for instance by getting students to write reports which are fully integrated into the course. Another is to exploit technology in all its aspects, for instance both to allow powerful visualisation, and to provide motivation.

We would point out that it is difficult to motivate properly unless it comes from within the student anyway; but it is easier to de-motivate! What we have to do is provide a positive environment, but given that the current generation of students is that which has been tested at every stage of their education as part of an unwise and destructive UK government strategy, the extrinsic motivation of an assessment which counts can sometimes be necessary to provoke serious mathematical activity!

The role of technology and other factors

Mathematics has four main ingredients (Challis and Gretton (2007)): symbols, numbers, words and images. Each contributes to effective understanding and communication. Representing the same mathematical concept in different forms helps in the process of making connections within and between topics, provides a mix which takes account of differing learning styles, and makes it more likely that the symbolic mathematics will be correct.

The strength of computers and graphic calculators is that they can be used to provide multiple representations of mathematical concepts quickly, correctly and easily. There does arise the question of which tools to use, but it is important first to consider what mathematical idea is being addressed, and the range of approaches to it, and then one can use whatever tools come to hand. Technology also works when used as a motivational aid. Using a technological aid, alternative what-ifs can be tested, incorrect working is easily discarded, feedback is virtually immediate and the student is enabled by knowing they are always correct!

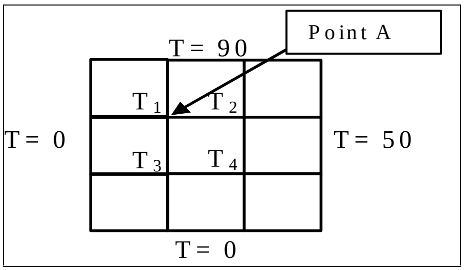
Indeed, the use of technology such as CAS, spreadsheets, and other visual technology does mean we all are enabled to get correct answers, validated by multiple means, but means we must develop wider skills. What do the results mean? How can we best communicate them to various different audiences? How do we convince others that we are right? These are essential meta-skills for engineers as for all of us.

We find we must develop a “rich” set of mathematical tasks. Ahmed (1987) describes these as ones which are accessible to all, allow challenges, get students to make decisions, get students to explain, reflect, interpret, and so on. Finally the task should be enjoyable!

Example

In Figure 1, we give an illustrative example of one part of a rich task, drawn from Challis and Gretton (1997): *"Well, here's another nice mes(h) you've gotten me into.”* (Apologies to Stan and Ollie.). It is one we use with first year engineering students as part of their learning to handle linear simultaneous equations. There are some points worth mentioning here. The task exploits commonly available technology, a spreadsheet. This enables students eventually to solve realistically large sets of equations. The problem involves a realistic context. The traditional order of the curriculum is disrupted once we allow use of technology. The problem requires some Symbolic manipulation; students can be required to give an Oral report at the end; the method is Numerical; and the technology allows easy Graphical presentation of results via the Excel plotting tools (Figure 2). Students can thus deploy a variety of learning styles in understanding and solving this problem. Some imagination must be applied to make the marking efficient: if a report section is to be included, then the purely instrumental parts of the marking must be streamlined as far as they can, and some indication of that is seen in Figure 1.

A flat, square plate as shown below has the temperatures on the edges held at the values shown:



You can use the discrete form of the energy conservation law, which says that the temperature at any point is the weighted average of the temperatures at the four surrounding points, to find the four equations necessary to give the approximate values of temperatures T_1 to T_4 at the points shown.

Now answer the questions below in the grid provided

	Your answer	
6. Write down in matrix form AT= B the equations satisfied by T_1 to T_4.		/4
7. Solve these equations using whatever technology you need, writing down here the value of T_1.	T_1 =	/4
8. Implement a strategy of mesh refinement as demonstrated in Excel in the lecture, to find the value of the temperature at the point A (which is the point where T_1 was originally specified) correct to 1 d.p., and write that value down here.	Temperature at point A to 1 d.p. =	/7

Figure 1

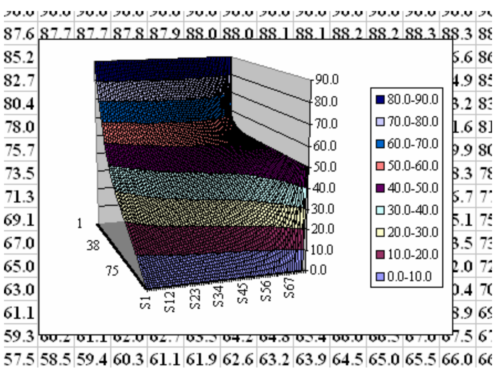


Figure 2

Conclusion

In a technology-rich environment, with multiple-representations and multiple independent ways of getting answers, arguably no student should ever get any sums wrong, in the narrow sense. But there is more than this, as mathematical learning is not only about getting right answers. Technological power can be used to motivate students, to allow the tackling of more realistic problems, and to vary the approach to learning, so that visual and verbal aspects, and active and reflective aspects, are all deployed in solving a rich problem.

References

Ahmed, A. (1987) *Better Mathematics*. London, HMSO.

Challis N.V. and Gretton H.W. (1997) Teaching Engineering Field Theory using IT. *Int J Mech Eng Educ,* Vol 25 (No 1): 1-12.

Challis N.V. and Gretton H.W. (1999) “Assessment: does the punishment fit the crime?”. In *Proc. ICTCM12,* San Francisco, Addison-Wesley- Longman.

Challis N.V. and Gretton H.W. (2007) *Fundamental Engineering Mathematics - A Student Friendly Workbook.* Ellis Horwood.

Challis N.V., Gretton H.W., Houston S.K. and Neill N. (2002) “Developing Transferable Skills: preparation fo remployment”. In P. Kahn and J. Kyle, eds. *Effective learning and teaching in Mathematics & its applications*. Kogan Page, pp 79-91.

Felder R.M. and Solomon B.A. (2008) “Learning Styles”. Accessed via http://www4.ncsu.edu/unity/lockers/users/f/felder/public/ILSdir/styles.htm (3rd March 2008).

Chinn, S. J. and Ashcroft, J. R. (1993) *Mathematics for Dyslexics: A Teaching Handbook.* Whurr Publishers Ltd (London).

HEA Engineering Subject Centre (2008) “Key Skills”. Accessed via http://www.engsc.ac.uk/er/key_skills/index.asp (3rd March 2008).

HM Treasury (2006) “Leitch Review of Skills: Prosperity for all in the global economy - world class skills”. Accessed via http://www.hm-treasury.gov.uk/newsroom_and_speeches/press/2006/press_leitch.cfm (3rd March 2008).

LTSN (2003) “Maths for Engineering and Science”. Accessed via http://www.mathstore.ac.uk/mathsteam/packs/engineering__science.pdf (3rd March 2008).

Support4Learning (2008) “Learning Styles”. Accessed via http://www.support4learning.org.uk/education/learning_styles.cfm (3rd March 2008).