Tree-structured Online Mathematics Exercises for Civil Engineering Students

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Abstract

In the 2012/2013 winter semester at RWTH Aachen University, a new type of e-learning exercises has been introduced into first-year mathematics courses for civil engineering students. The project is intended to increase the amount of feedback given to learners – and thus improve their maths exam performance, which had been noticeably declining over the past years. The exercises are created as Moodle lessons. A single lesson covers one particular task, for example a principal component analysis of a given matrix. Via sequentially linked pages, users are guided through the solution process. Depending on their answers to questions, they are forwarded to different further pages. The resulting tree structure allows for adaptive, individual feedback. During the first year of use, the effectiveness of the exercises has been evaluated by a survey and a statistical analysis. Results have turned out positive, and the exercises continue to be used.

Motivation for developing new exercises

Over the past years, first-year mathematics lectures for civil engineering students at RWTH Aachen University have basically stayed constant in their choice of topics and exam tasks (calculus, linear algebra, differential equations etc.). At the same time, the students' exam performances have been noticeably declining ratio-wise. While this may or may not be caused by a change in first-year students' qualifications, measures were sought to counteract this development. High application numbers for civil engineering programmes – with roughly 1200 students registered for one maths lecture – have further contributed to a need for change in the way material is presented to students.

The mathematics lecture itself is supplemented by a large range of various learning activities. A major issue that arose was that most of the activities offered before winter 2012 lacked individual feedback. This was found to be a disadvantage to students, in particular those in the big group of average students. For example, presence-based supplementary courses are still largely based on frontal presentation of possible solutions. While such sample solutions are certainly useful, they leave learners passively receiving instead of actively "doing the maths" themselves. Learning mainly from sample solutions may be sufficient for good students, but it possibly demands too much of average learners. Due to a lack of direct feedback, it is further conceivable that many students did not recognise their own misconceptions and underdeveloped solving skills until they had received their exam results. Efforts for introducing autonomous student work into presence-based courses were and are made, but big group sizes of 100 or more leave these endeavours mostly cumbersome and unrewarding.

During the reading period, learners can work on special homework exercises (which are similar to exam tasks) and hand in their written solutions. These are then marked and commented on by teaching assistants, and finally given back to the students. The service is used by about 400 out of the approximately 1200 students per semester. While it is an effective means of feedback, this method has its drawbacks: apart from the associated workload of checking and commenting hundreds of written solutions, it offers no help to the students on how to exactly solve the exercises. For a large number of students, the exercises are too much to handle. Many learners have trouble approaching complex mathematical tasks in a rational, target-oriented way. Accordingly, many of the solutions handed in are far from acceptable. These students probably need comprehensible, small-step coaching – instead of frontally presented sample solutions – in order to learn how to solve exam-relevant tasks.

Simple online assignments have been introduced even before the exercises described in this paper. Students must still solve about half of these assignments correctly as a requirement for exam application. They are realised as Moodle quizzes (see Quiz module (2012)). Such a quiz consists of a couple of questions which are randomly selected from a question pool, and which are usually easier and less complex than exam tasks. Learners have to first solve on paper the assignments given in the questions, and then enter the correct results into the corresponding text boxes or as multiple choice answers. The only feedback here is "wrong" or "correct", there are no other responses or even progression changes that depend on learner actions.

Thus, two important demands concerning the new exercises' design were that they should provide individual feedback and offer coaching whenever the students might need it. Moreover, the high number of registered students suggested e-learning exercises to be the most practical approach.

Exercise design as tree-structured Moodle lessons

After looking through various e-learning platforms, Moodle was chosen as the one to use for implementing the e-learning exercises (see Moodle (2014) – the exercises currently run on version 2.3.1). A crucial factor for this decision was the "lesson" feature that Moodle offers (see Lesson module (2012)). It allows questions which – after a user has submitted an answer – automatically link to different further pages depending on the answer given. This means that the progression of a lesson can change according to the user's answers, making individual feedback possible. The mathematics exercises were thus devised as Moodle lessons. One exercise always corresponds to one particular task; the idea is to accompany users through the whole solution process. Starting with a task description, users are guided along sequentially linked pages until the exercise is finished and the task is completed. At numerous points, user input is required, which can change the path on which a user reaches the end of a lesson. The resulting exercise structure is vaguely linear, but with "path forks" at user input points. It can intuitively be visualised as a tree structure. (This is only a rough intuition. There are in fact no restrictions as to how pages can be linked with each other, and cycles are possible. Strict tree structures exist only locally.) An example structure for a curve sketching exercise is given in figure 3 at the end of this section.

It is important to note that this tree structure creates a kind of adaptivity, as the exercises do change behaviour automatically to suit user needs (for the concept of adaptivity see Oppermann (1997)). While they do not intelligently keep track of user attributes, the exercises immediately send users to appropriate further paths after input points. This can be called *local adaptivity.*

On a technical level, there are two basic page types in lessons: Content pages and question pages. Content pages consist of a body containing text, formulas, images etc., and of one or more link buttons at the bottom which link to different pages.

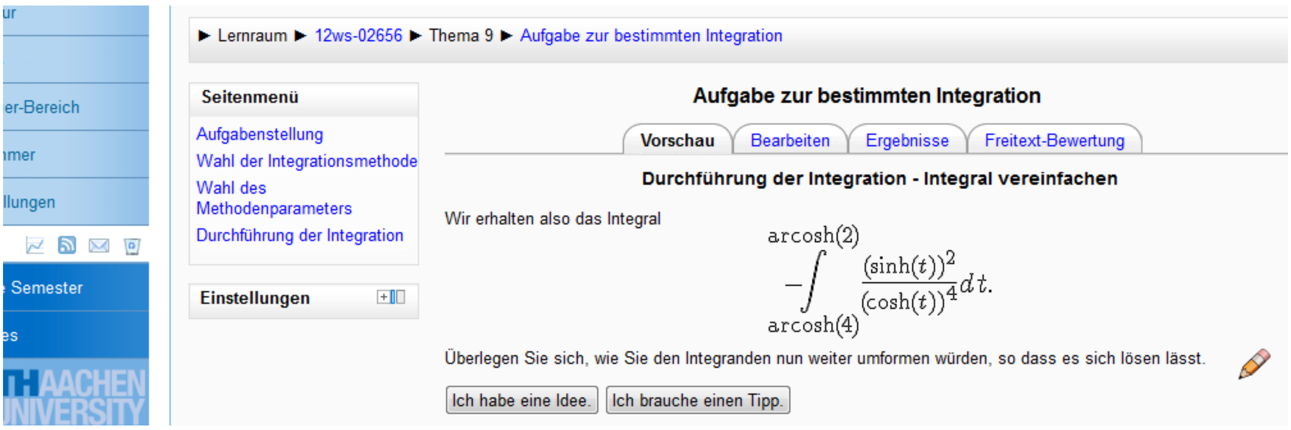

Figure 1. Content page in an exercise on integration – note the menu and pencil symbol.

For example, content pages might explain task-related thoughts, identify the next step, demonstrate sample calculations, or visualise a situation graphically. With these elements, coaching can be realised. Using two or more link buttons at the bottom, it is also possible to let the user decide how to continue: Go directly to the next subtask, or use an auxiliary path that offers tips or advice. An example is shown in figure 1 on the previous page. Unlike content pages, question pages always require input from the user, and can again be subdivided into different types. Question types used in the exercises are multiple choice, short answer (typing an answer into a text box) or matching (correctly associating entries from one list with entries from another). Participants' answers can be used to diagnose misconceptions or calculation errors, and to direct participants to explanation pages accordingly. As solving the exercises should also train students in working autonomously, there are some other design elements which are meant to improve learning behaviour and thorough argumentation. Graphical symbols at the right page margin tell users when they are expected to write/calculate on a piece of paper on their own (pencil symbol), to look up a mathematical concept (book symbol), or to pay close attention to pitfalls (lightbulb symbol). A menu at the left page margin shows important content pages as clickable links. It allows quick navigation through the exercise and acts as a reminder of major solution steps.

Content-wise, the exercises' designs are based upon typical exam tasks. Lessons developed up to now cover the following tasks:

- Sketching a set of complex numbers given by an inequation
- Solving an inequation of real numbers
- Convergence of a sequence and finding its limit
- Convergence of a series
- Sketching the curve of a univariate real function, which includes roots, relative extrema etc.
- Integrating a univariate real function
- Checking a univariate real function for continuity and differentiability at a specific point
- Solving a linear system
- Normalising and visualising a quadric, which includes a principal component analysis of the corresponding symmetric matrix
- Solving a Bernoulli differential equation

From winter 2012 onwards, the exercises have been made available on the standard online platform of RWTH Aachen University – the so-called L²P. As soon as the mathematics lecture reaches a certain topic, corresponding exercises are released for registered students. So far, usage of the exercises has always been fully voluntary.

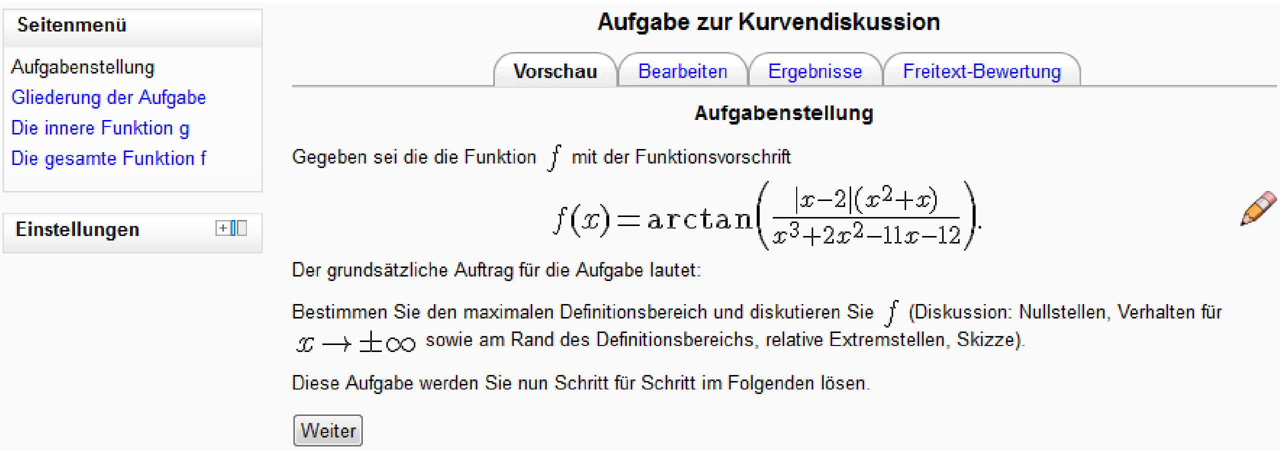

Figure 2: Task description page in an exercise on curve sketching. The structure of the entire exercise can be seen on the next page.

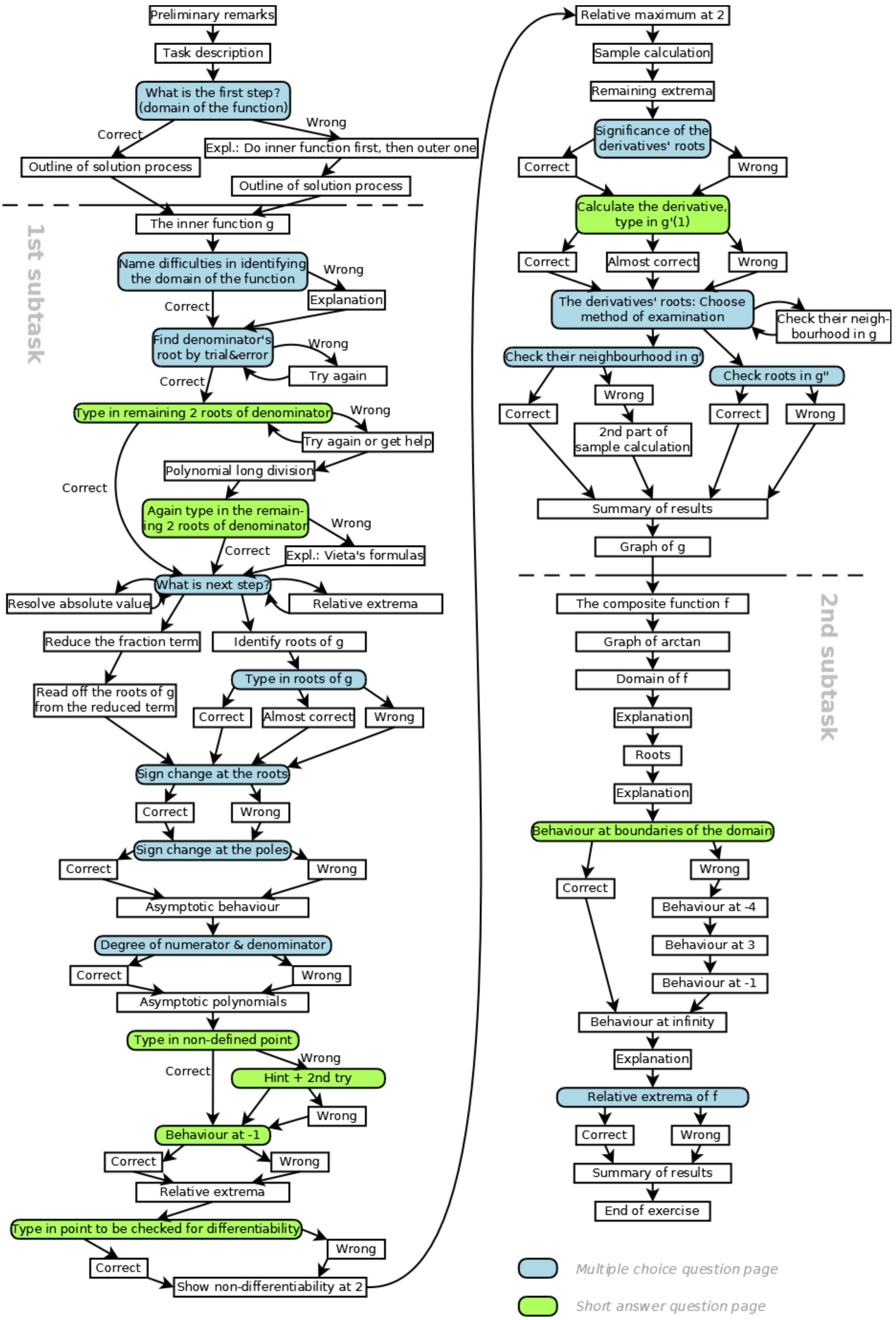

Figure 3. Structure of the exercise on curve sketching.

Assessment of the exercises' effectiveness

Despite being purely voluntary, the exercises have been used by about 45% of 1256 registered students in winter 2012/2013 and by about 35% of 1120 registered students in summer 2013. Exam results in winter 2012/2013 showed some improvement, and in summer 2013, grade distribution and pass rates were significantly better than in the year before. This does not necessarily have to be caused by the exercises' introduction, however. So in order to further assess the impact of the exercises during this first year of use, two methods were utilised: A questionnaire asked students for their opinion, and a statistical analysis investigated the association between exercise usage and exam grade. The results of both methods have turned out positive.

The questionnaire was handed out in a lecture session and also made available online on the L²P platform. It contained questions regarding thoroughness of usage, the exercises' general helpfulness, helpfulness for exam preparation in particular, and appropriateness of exercise design. 270 of around 500 survey participants stated that they had actually used at least one exercise; this group is used as basis for the following percentages. Numbers are rounded to the nearest integer.

- 69% have stated that they have used most or all of the exercises offered, and 51% said they have actually carried out calculations on paper when a lesson page asked them to.
- 67-83% found that the exercises helped them to better understand lecture contents, to apply those contents in mathematical tasks, to see solution structures as a whole, to compose written solutions, and to assess one's own level of knowledge.
- 58% have stated that the exercises were helpful for exam preparation, and 37% thought that without the exercises they would have performed worse in the exam. However, 52% still found the exercises to be too different from actual exam tasks.
- 74-82% found the exercises appropriate in terms of difficulty, length, step width and amount of coaching, respectively.

For the statistical analysis of association between exercise usage and exam grade, a table was set up for each of the two semesters in question (winter 2012/2013 and summer 2013). Rows containing the anonymous student data were combined with columns containing the corresponding number of exercises used and the corresponding exam grade. Correlation and contingency coefficients between exam grade and number of exercises used could then be determined. For the sake of brevity, we only present the findings for summer 2013 in this paper; results for winter 2012/2013 are similar. Numbers are rounded to the nearest hundredth. 683 students sat the exam in summer 2013, which form the basis for the following calculations. Distinguishing between students who have or have not used at least one exercise, and between students who have or have not passed the exam, the corrected contingency coefficient is 0.55. The contingency table is as follows:

Summer 2013	Exam passed	Exam not passed	Total
Has used exercises	296	49	345
Has not used exercises	154	184	338
Total	450	233	683

The Pearson correlation coefficient between exam grade and number of exercises used is -0.43 for summer 2013. Note that German grades go from 1 (excellent) to 5 (not passed), hence the negative sign. Using a p-value test, this result can be shown to be statistically significant. A point cloud, as a way to illustrate the association, has turned out uninformative because of discrete values. However, by visualising average grades depending on the exact number of exercises used, a sufficiently informative illustration can be achieved, seen in figure 4 on the next page.

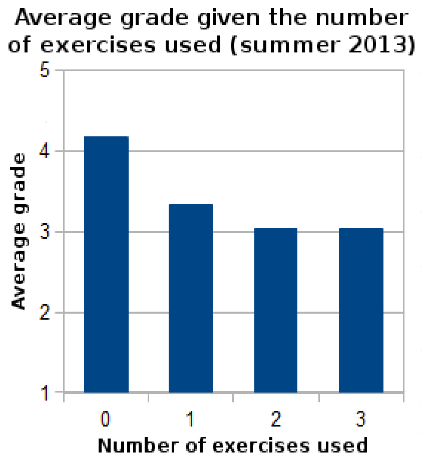

Figure 4. Association between exercise usage and exam grade.

Taking into consideration that more than 50% of students have received a grade between 5 to 3.7, it can be concluded that – on average – using the exercises has brought students to substantially higher rankings in the whole cohort. As a comparison, one can determine the correlation coefficient between exam grade and number of mandatory online quiz assignments solved. This coefficient is -0.19 for summer 2013 (again negative), suggesting a less strong association than with the exercises described in this paper.

Finally, it should be checked if higher exercise usage is merely an indicator for higher general diligence. As a presumably valid indicator for diligence, we used the amount of mandatory online quiz assignments solved: They test rather than coach, and students only need 50% of them for exam application, so learners who solve more than the required amount can be considered diligent. Percentages of diligent learners among all 683 exam participants and among exercise users in particular both depend on the exact threshold value used. Using various thresholds like 60% or 80% of quiz assignments solved, it has turned out that the proportions of diligent students in the two groups (among all exam participants or among exercise users only) never differ by more than about 8% in summer 2013. We can thus suspect that the above correlation is not significantly distorted by diligence traits.

Conclusion

While the new exercises did not solve all problems, their effects were markedly positive. This includes student opinions as well as effects on examination success. For this reason, the exercises continue to be used in mathematics courses for civil engineering students and have at least partly replaced written homework. The tree structure of the exercises has introduced an element of adaptivity which, up to now, we could not find in any other e-learning system.

Concerning future developments, there are still some aspects to be worked upon. Exercises do not yet cover all relevant topics, so more should be developed especially for the summer course. Because of their strong association with exam success in comparison to the online quiz assignments, it is also worth considering to make the exercises mandatory in some way. A reasonable method of validation would then be needed. For more extensive comparisons, it will be sensible to also determine the effectiveness of other learning activities offered, in particular written homework assignments. And lastly, in order to have a reasonable number of tests, more student data will be evaluated in forthcoming semesters.

References

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